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EVALUATION OF THE CONCENTRATION  
METHOD FOR DETERMINING THE V/L RATIO  
IN TWO-PHASE FLUID FLOW

A THESIS

Presented to  
the Faculty of the Graduate Division  
by  
Gordon Edward Johnson III

In Partial Fulfillment  
of the Requirements for the Degree  
Master of Science in Chemical Engineering

Georgia Institute of Technology

June, 1961

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Date Approved by the Chairman: May 17, 1961

## ACKNOWLEDGMENTS

The author wishes to acknowledge Dr. H. C. Ward for his suggestion of the problem.

The author's appreciation is extended to his wife for her patient assistance and encouragement.



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## NOMENCLATURE

A	cross-sectional area of pipe, $\text{ft.}^2$
a	constant defined by equation (22), lb. solute . cc/lb. solution . gm
b	constant defined by equation (22), lb. solute/lb. solution
C	concentration, in liquid phase, lb. solute/lb. solution
$C_o$	concentration based on total mass flow rate, lb. solute/lb. solution
D	inside diameter of pipe, ft.
E	potential measured at thermocouples, mv.
$g_c$	conversion factor, $32.17 \text{ ft. lb. mass/sec.}^2 \text{ lb. force}$
$h_s$	scale reading of solute rotameter, dimensionless
K	ratio defined by equation (10), dimensionless
L	length, ft.
m	mass, lb.
N	normality, moles/liter
$N_{RE_1}$	liquid Reynolds number based on the pipe diameter, dimensionless
P	pressure, psi
$Q_g$	gas flow rate as read from calibration curve for gas rotameter before correcting the variations from standard conditions, CFM
$Q_l$	volumetric flow rate of liquid, GPM
R	void fraction of phase in pipe, dimensionless
t	temperature, $^{\circ}\text{C}$



$T$	temperature, $^{\circ}\text{K}$
$(V/L)_c$	ratio of volumetric flow rate of gas to volumetric flow rate of liquid as determined by the concentration method
$(V/L)_m$	ratio of volumetric flow rate of gas to volumetric flow rate of liquid as determined by metering the separate phases
$\bar{V}$	average velocity of phase, ft./sec.
$v$	specific volume, cu.ft./lb. mass
$W$	mass flow rate, lb. mass/sec.
$x$	flowing quality defined by equation (4)
$y$	static quality defined by equation (5)
$\mu$	viscosity, centipoise
$\rho$	density, gm/cc

#### SUBSCRIPTS

$c$	refers to measurement by concentration method
$g$	refers to gas phase
$l$	refers to liquid phase after addition of solute
$m$	refers to measurement by metering phase(s)
$s$	refers to solute at point of injection
$t$	refers to test section
$w$	refers to liquid phase prior to addition of solute

## SUMMARY

In the two-phase flow of a liquid and a vapor, one of the important variables that characterizes the flow is the ratio of the volumetric flow rate of vapor to the volumetric flow rate of liquid, designated here as  $V/L$ . While the  $V/L$  ratio may be determined by metering the phases separately, this is not possible for systems with mass transfer between the phases. A suggested method for determining the  $V/L$  ratio is based on the continuity of a non-volatile solute in the liquid phase and involves the measurement of its concentration during single-phase flow and measurement of its concentration, temperature and pressure at the point at which the  $V/L$  ratio is desired. A knowledge of the thermodynamic properties of the fluid is presumed.

This investigation is concerned with the evaluation of the "concentration" method on an air-water system at water flow rates from 0.7 to 50 GPM and air flow rates from 0.004 to 0.09 pounds per second using sodium hydroxide as the solute. The experimental apparatus provided means of metering the air, water and solute into a 1 1/2-inch test section where the  $V/L$  ratio was determined.

It was found that the  $V/L$  ratio could be determined with relatively good accuracy over a narrow range of flow conditions. The accuracy in the  $V/L$  ratio is directly pro-

portional to the accuracy in measuring the concentration and inversely proportional to the flowing quality. The accuracy in measuring the temperature and pressure also affects the measured V/L ratio. The range of applicability is determined largely by the greatest error that may be tolerated in the V/L ratio and by the accuracy to which the concentration may be determined.

Observations were made when a dye was injected into the two-phase flow in a glass pipe. It was observed that for a steady injection rate the dye was not distributed evenly in the liquid phase except in the cases of annular, stratified, or bubble flow. As irregularities appeared in the flow, the dye became evenly distributed in the liquid phase. These axial gradients were observed in some cases as far down the pipe as 40 feet where the fluids were discharged to the sewage system. Because of this and inconsistencies found in the data when the solute was injected during two-phase flow, it was concluded that the solute should be injected during single-phase flow.

The need of a continuous method for monitoring the concentration in the liquid stream is pointed out. The single-phase mixing length was determined as a function of the Reynolds number within a narrow range of variables to illustrate the magnitude of the mixing length necessary prior to the commencement of two-phase flow. Future work is urged



on a system with mass transfer between the phases such as the steam-water system.

## CHAPTER I

### INTRODUCTION

In the two-phase flow of a vapor and a liquid, one of the important variables necessary to characterize the flow is the ratio of the volumetric flow rate of the vapor to the volumetric flow rate of the liquid, designated here as  $V/L$ . It may be noted from the literature that the  $V/L$  ratio is one of the variables used for estimating two-phase pressure drops. The variable  $X^2$  of Lockhart and Martinelli (1) is a function of the  $V/L$  ratio and other variables. Armand and Tretchev (2) based their pressure-drop correlation on the single-phase pressure drop and the volume fraction of gas in the pipe,  $R_g$ , which is a function of the  $V/L$  ratio and other variables. Ward, Goglia, et.al. (3) in evaluating a JP-4 aircraft system have shown that the performance depends on the  $V/L$  ratio, the operating pressure and temperature and the dynamic flow pattern.

The  $V/L$  ratio may be determined for systems with no mass transfer between the phases from a knowledge of the mass flow rate of each phase, the temperature and pressure of the fluids, and the thermodynamic properties of the fluids. In systems of this type the mass flow rates of each phase can usually be metered and there are few complications in determining the  $V/L$  ratio. However, in systems with mass

transfer between the phases such as evaporating flow or condensing flow in a tube, the V/L ratio is not so simply determined. At present there is no known way of directly measuring the V/L ratio in systems with mass transfer between the phases. Because of the increased emphasis in the field of cryogenics, especially cryogenic fuels, a great deal of work has been done of late on two-phase flow. Nuclear reactor technology and power generation from reactors have also caused a need to better understand the basic mechanisms of the phenomena of two-phase flow. In industrial operations involving evaporation, flashing, condensation and evolution of condensed gases, there is a need for better knowledge and fundamental concepts to facilitate design of new equipment and to enable evaluation and refinement of existing facilities. In most of these operations and in the research being carried on in two-phase flow there is a need for a method of directly measuring the V/L ratio.

Ward, et.al. (4) have suggested two indirect methods for measuring the V/L ratio based on the work of Dengler and Addoms (5) and Linning (6). One method involves injecting a non-volatile solute, soluble in the liquid phase, into the flow stream. By injecting at a known rate with respect to the total mass flow rate and by measuring the temperature, pressure, and concentration at a point downstream, the V/L ratio at that point may be calculated. The other method involves measuring the temperature, pressure, total mass flow

rate, the volume fraction of liquid in the pipe, and the flow rate of momentum. The V/L ratio may be computed if these quantities are known at the point in question.

To provide the background for these two methods, consider a two-phase system at steady state, vapor and liquid, with the fluids flowing co-currently. The continuity equation may be written for each phase.

$$\bar{V}_g A_g = W_g v_g \quad (1)$$

$$\bar{V}_l A_l = W_l v_l \quad (2)$$

Dividing equation (1) by equation (2) gives

$$\frac{\bar{V}_g A_g}{\bar{V}_l A_l} = \frac{W_g v_g}{W_l v_l} \quad (3)$$

The flowing quality,  $x$ , is defined as

$$x = \frac{W_g}{W_g + W_l} = \frac{W_g}{W} \quad (4)$$

and the static quality,  $y$ , is defined by

$$y = \frac{m_g}{m_g + m_l} = \frac{\frac{A_g}{v_g} dz}{\frac{A_g}{v_g} dz + \frac{A_l}{v_l} dz} = \frac{A_g}{A_g + \frac{v_g}{v_l} A_l} \quad (5)$$

From equations (3), (4) and (5) the following relations are defined:

$$W_g = x W \quad (6)$$

$$W_l = (1 - x) W \quad (7)$$



$$A_g = \frac{\frac{V_g}{V_L} \cdot \frac{y}{1-y} A}{1 + \frac{V_g}{V_L} \cdot \frac{y}{1-y}} \quad (8)$$

$$A_L = \frac{A}{1 + \frac{V_g}{V_L} \cdot \frac{y}{1-y}} \quad (9)$$

If one defines the ratio of the velocities,  $K$ , as

$$K = \frac{\frac{V_g}{V_L}}{\frac{y}{1-y}} = \frac{\frac{x}{1-x}}{\frac{y}{1-y}} \quad (10)$$

then using equations (3), (6), (7) and (10) the  $V/L$  ratio may be expressed in the following forms.

$$\frac{V}{L} = K \frac{A_g}{A_L} \quad (11)$$

$$\frac{V}{L} = \frac{V_g}{V_L} \cdot \frac{W_g}{W_L} \quad (12)$$

$$\frac{V}{L} = \frac{V_g}{V_L} \cdot \frac{x}{1-x} \quad (13)$$

It may be noticed from equation (10) that only when the velocities of the phases are equal is the static quality equal to the flowing quality.

In the concentration method of determining the  $V/L$  ratio, a non-volatile solute is dissolved in the liquid phase or a substance not soluble in the liquid phase is added

to the gas phase. Consider that a solute is dissolved in the liquid phase prior to the commencement of two-phase flow such that the concentration in the liquid is  $C_0$  mass fraction at the point of injection. At a point downstream where two-phase flow exists the concentration of solute in the liquid phase will be  $C$ . By a material balance one obtains, for steady state,

$$C_0 W = C W_L \quad (14)$$

The continuity equation gives

$$W = W_L + W_g \quad (15)$$

Combining equations (14) and (15) gives

$$\frac{W_g}{W_L} = \frac{C}{C_0} - 1 \quad (16)$$

By substituting equation (12) into equation (16), an expression for the V/L ratio is derived.

$$\frac{V}{L} = \frac{v_g}{v_L} \left( \frac{C}{C_0} - 1 \right) \quad (17)$$

The main disadvantage of this method is the contamination of the fluid but the variables which must be known are more difficult to measure. In order to provide a comparison of the two methods and to be able to understand the merits of each method a brief outline of the "momentum" method is helpful.

The total momentum flow rate past a section is given by

$$M = \frac{W_g \bar{V}_g}{g_c} + \frac{W_l \bar{V}_l}{g_c} \quad (18)$$

Using the continuity equations (1) and (2) and equations (6) and (7), M may be expressed as

$$M = \frac{v_g x^2 W^2}{g_c A_g} + \frac{v_l (1-x)^2 W^2}{g_c A_l} \quad (19)$$

Using equations (8) and (9) and defining a as

$$a = \frac{v_g}{v_l} \cdot \frac{y}{1-y} = \frac{A_g}{A_l} \quad (20)$$

equation (19) can be solved for x, the flowing quality. The result is

$$x = \frac{a v_l \pm \sqrt{\frac{a}{1+a} \left[ \frac{g_c A M}{W^2} (v_g + a v_l) - v_g v_l (1+a) \right]}}{v_g + a v_l} \quad (21)$$

Then by using the value of x so obtained the V/L ratio may be determined from the expression

$$\frac{V}{L} = \frac{v_g}{v_l} \cdot \frac{x}{1-x} \quad (13)$$

This method involves the measurement of the temperature, pressure, total mass flow rate, the volume fraction of the

pipe occupied by the liquid, and the total momentum flow rate at the point in question. While this method involves more measurements and is more complex, it appears useful for a wider range of systems. Although it will take considerable ingenuity to design a device for the measurement of the momentum with a minimum disturbance of the flow pattern, this method does have the immediate advantage of preventing contamination of the fluid in question. The concentration method should find an immediate application as an aid in the development of the momentum method.

It may be noted that the simultaneous determination of the V/L ratio and the volumetric fraction of liquid in the conduit,  $R_1$ , may be used to calculate the average velocities of the phases. Several investigators have measured the volumetric ratio for various systems. Dengler and Addoms (5) measured  $R_1$  based on the development work of Lee (7). Isbin, et.al. (8), in determining the ratio of the velocities of the two phases, measured the volumetric fraction of gas in a pipe and reviewed the known methods of measurement.

The complexities that arise in studying and characterizing two-phase flow may be better understood by a qualitative examination of the flow regimes that can exist. The flow patterns do not change from one pattern to the next at any well-defined transition point, but rather over a comparatively wide range of flow variables. These flow patterns depend on the relative amounts and velocities of



the phases, the nature of the phases such as viscosity and density, the geometry of the piping, entrance effects and external forces acting on the fluid.

Although different investigators have used varying nomenclature for the regimes, the following terminology will be used throughout this report. Seven basic flow patterns have been observed in the horizontal co-current flow of a liquid and a gas. When starting with a horizontal pipe running full of a liquid and adding increased amounts of the vapor phase, they are:

1. bubble flow, in which bubbles are entrained in the liquid and move at approximately the same velocity as the liquid;

2. stratified flow, in which the gas occupies the upper portion and the liquid the lower portion of the pipe with a smooth interface between phases;

3. wave flow, in which the smooth interface becomes disturbed by waves;

4. plug flow, in which the waves fill the total area of the pipe cross-section such that large plugs of gas move along the pipe length with the liquid phase controlling;

5. slug flow, in which rapidly moving slugs of liquid move along the pipe length with the gas phase controlling;

6. annular flow, in which a high velocity core of gas forces the liquid around the periphery of the tube wall in the form of an annulus; and

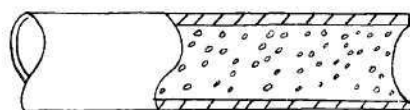
7. mist flow, in which drops of the liquid are dispersed throughout the continuous gas phase.

It should be pointed out that although these basic patterns exist, the transition from one regime to another is not smooth and various combinations of these patterns are possible, such as slug-annular and wave-annular.

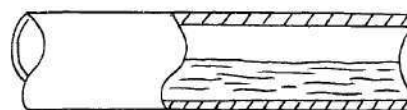
Figure 1 contains sketches of the various flow patterns. All of the basic flow patterns except mist flow, as well as several combinations, were observed in this study. Parrot (9) on the basis of 647 experimental runs observed the flow regimes shown in Fig. 2. These observations were made on the apparatus used in this investigation and his findings are reproduced for reference and clarity. The visual observations made during this investigation agreed with Parrot's findings.

From the foregoing discussion it appears that the momentum method has, in principle, considerable merit to recommend the development of a practical method for measuring the  $V/L$  ratio based on this principle. As an experimental tool, the concentration method has utility in this pursuit, specifically as a check on the accuracy of the momentum method.

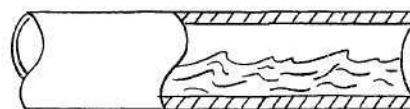
The purpose of this investigation, then, was to evaluate the concentration method in a system without mass transfer to determine its limitations, the degree of accuracy obtainable, and, insofar as possible, specify and furnish the data



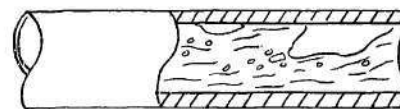
a. Bubble



b. Stratified



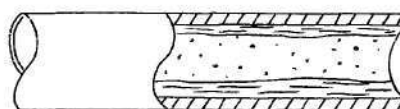
c. Wave



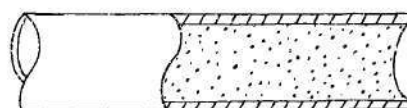
d. Plug



e. Slug



f. Annular



g. Mist

Figure 1. Two-Phase Flow Patterns  
in a Horizontal Pipe

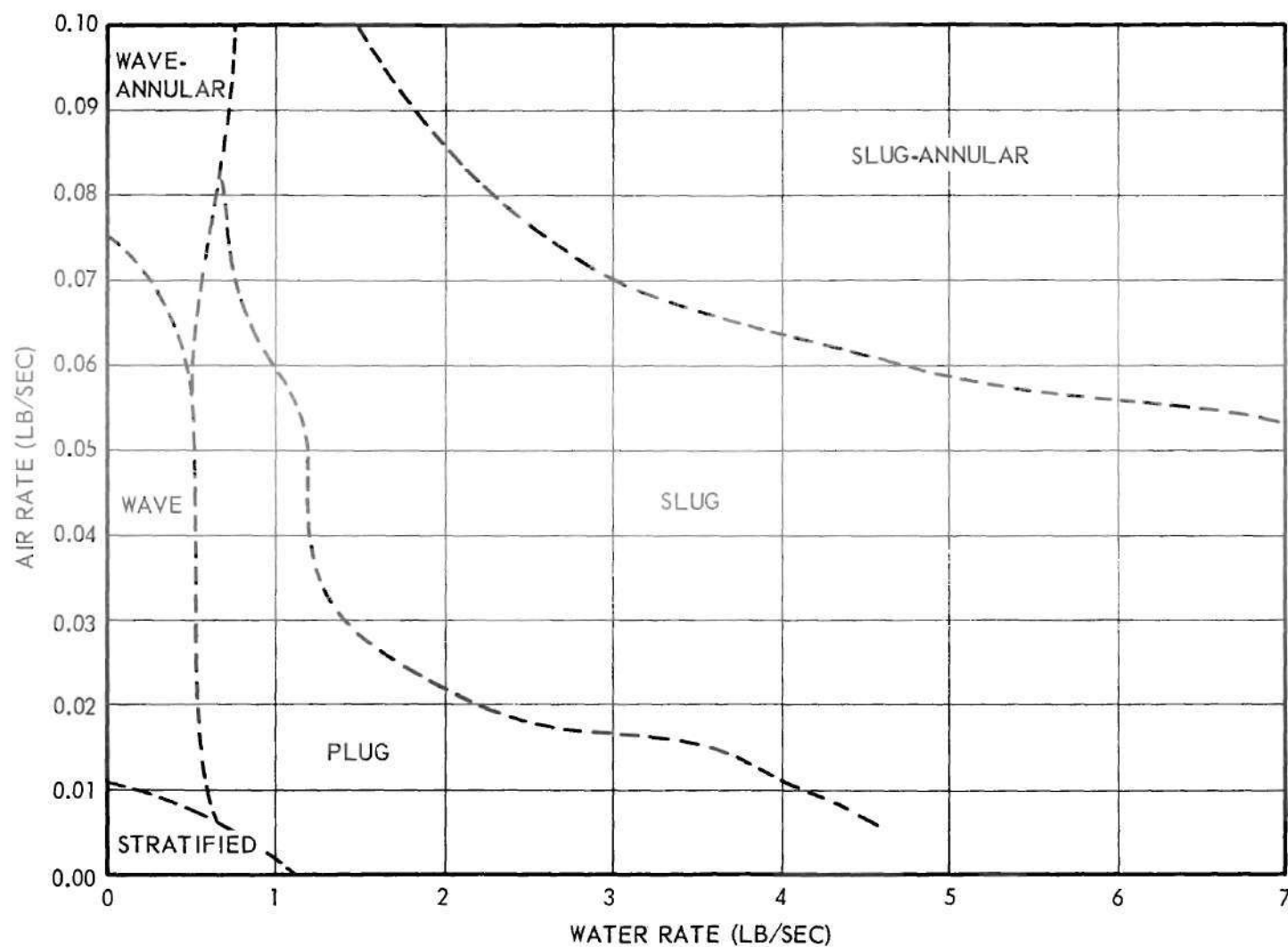


Figure 2. Observed Flow Patterns for Co-Current Flow of Air-Water Mixtures in 1 1/2-Inch Pipe as a Function of Air and Water Mass Flow Rates. (Parrot (9))



necessary to make the method practicable to other systems.

The experimental phase consisted of determining the V/L ratio by the concentration method and comparing this with the metered V/L ratio to establish the limits of accuracy. Associated problems such as injection methods, mixing lengths, sampling techniques, and range of applicability were also determined approximately.

A preliminary study was made to determine the solute type, sampling techniques and the effects of measurement errors in the variables determining the V/L ratio.

## CHAPTER II

### PRELIMINARY STUDY

Prior to the commencement of the experimental work, several problems were taken under advisement. The solute type, method of injection of the solute, and the analytical method of determining the concentration of the solute during two-phase flow were considered to make the concentration method as generally applicable as possible to other systems and to obtain optimum results from this investigation. In determining the solute to be used, it was decided that the solute should have certain characteristics. The solute should be:

1. non-volatile;
2. non-corrosive;
3. easily analyzed for; and
4. easily characterized, i.e., the thermodynamic and physical properties should be available.

In addition to these four primary requirements, the toxicity and cost of the material must also be considered. Requirements (1) and (3) tend to eliminate most organic compounds, especially because of the difficulty of the quantitative detection of such compounds. The methods of determining the concentration of the solute must be considered concurrently since the ease and rapidity of concentration

measurements are important. Several general techniques were considered, foremost among which were methods involving measuring the photoconductivity of a dye solution, the electrical conductivity of an electrolyte, the refractive index, the density, or the normality by titration. The purity of water from the city water supply was not such that one could consider measuring the electrical conductivity, the refractive index, or the density with any desired degree of accuracy. Because the water storage tank was open to the atmosphere and because of the oxidation of the steel pipes in the apparatus, the water would undoubtedly be contaminated rendering the latter three methods useless. The determination of a dye in solution was briefly considered (Appendix IV) and rejected. Titration appeared to be the most appropriate method and consequently it was used in this study.

Titration methods are most accurate when an inorganic salt is used, but the chief disadvantage in using an electrolyte is that they are generally very corrosive. The more common electrolytes were reviewed and it was noted that mild sodium hydroxide solutions have little corrosive effect on steel. Physical and thermodynamic data are widely published on aqueous sodium hydroxide solutions. Viscosity data over the range of this investigation are given in the International Critical Tables (10). These data are reproduced in Appendix III. Data on the temperature-density-concentration relationship are given in the Chemical Engineers' Handbook (11).

These data were fitted to an equation of the form

$$C = ap - b \quad (22)$$

where the constants  $a$  and  $b$  are functions of temperature and are given in Appendix III.

The only requirement imposed on the method of injecting the solute into the flow system was that the injection be such that the rate of addition of solute be continuous and steady. This was accomplished by imposing a constant pressure on the solute reservoir and adjusting the flow by a valve.

Ideally, the sampling method would be one for which the fluid was not disturbed in the pipe and the analysis carried out continuously. There are commercial instruments that could be adapted to continuously monitor the concentration, however, the expense was prohibitive. In sampling the fluid at the test section, the fluid was withdrawn from a point on the periphery of the pipe such that the sampling rate was a very small percentage of the mass flow rate of the phase being sampled.



### CHAPTER III

#### INSTRUMENTATION AND EQUIPMENT

The apparatus used in this study was originally built by Ward (12) and has been used subsequently by Sharp (13), Gossage (14) and Parrot (9). The equipment was modified slightly for each study and as a consequence contains components that were not used in this investigation. Only the elements essential to the experimental phase of this study will be discussed.

A schematic diagram of the apparatus is shown in Fig. 3. Photographs of the equipment are shown in Figs. 4 through 9.

The apparatus consisted of, in its essential features, an air reservoir and air metering devices, a water supply tank, pump, flow meters, a mixing section where the air and water were brought together to establish a definite flow pattern, a test section at which point the V/L ratio was determined, and a solute injection system whereby a metered amount of solute could be added to the flowing fluids. A section was installed for measuring the two-phase mixing length and solute injection points were provided for injecting a dye into the stream so that mixing effects could be observed in the glass pipe. A detailed discussion of these components and associated equipment is given in the fol-

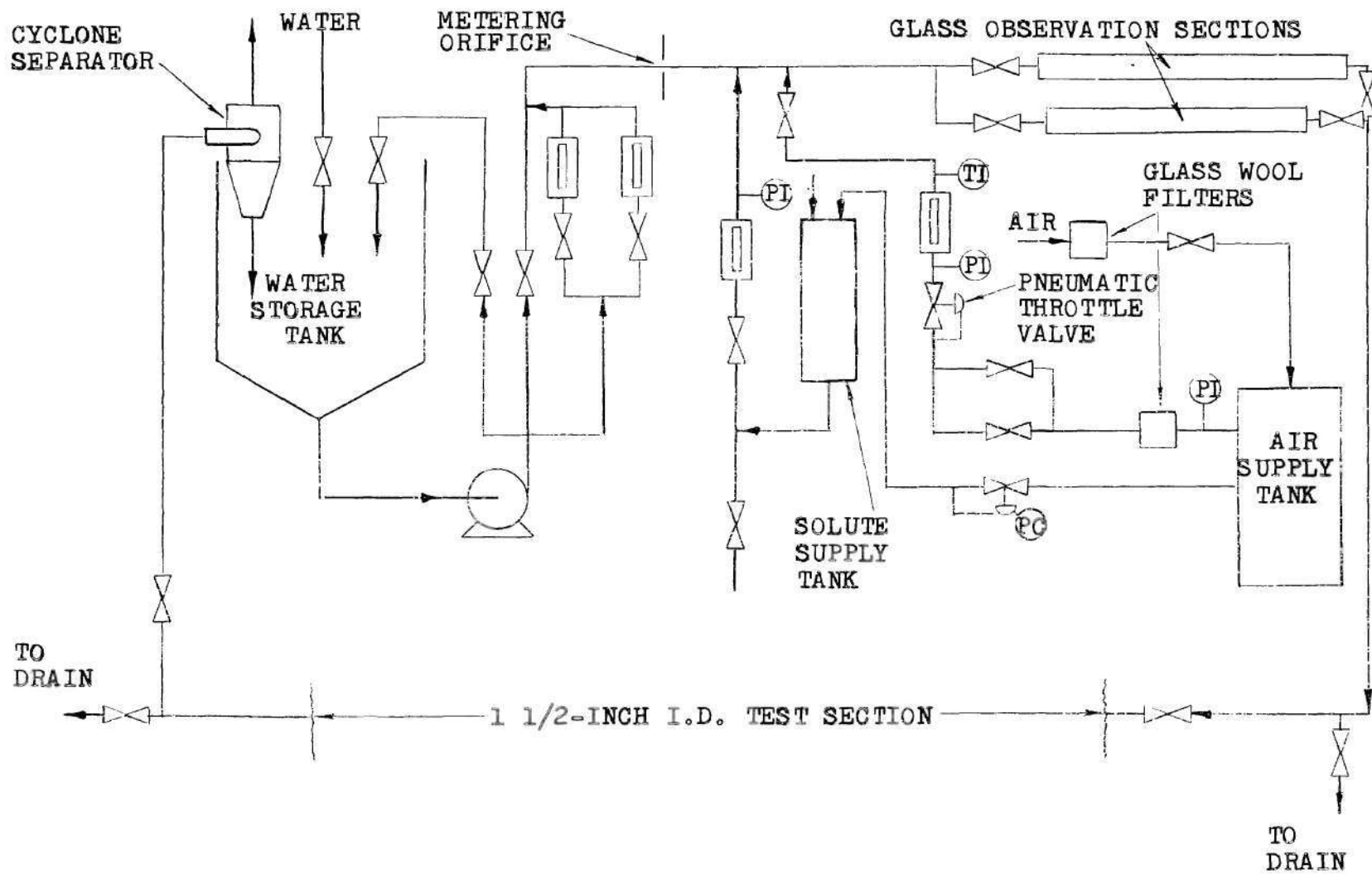


Figure 3. Diagram of Experimental Apparatus

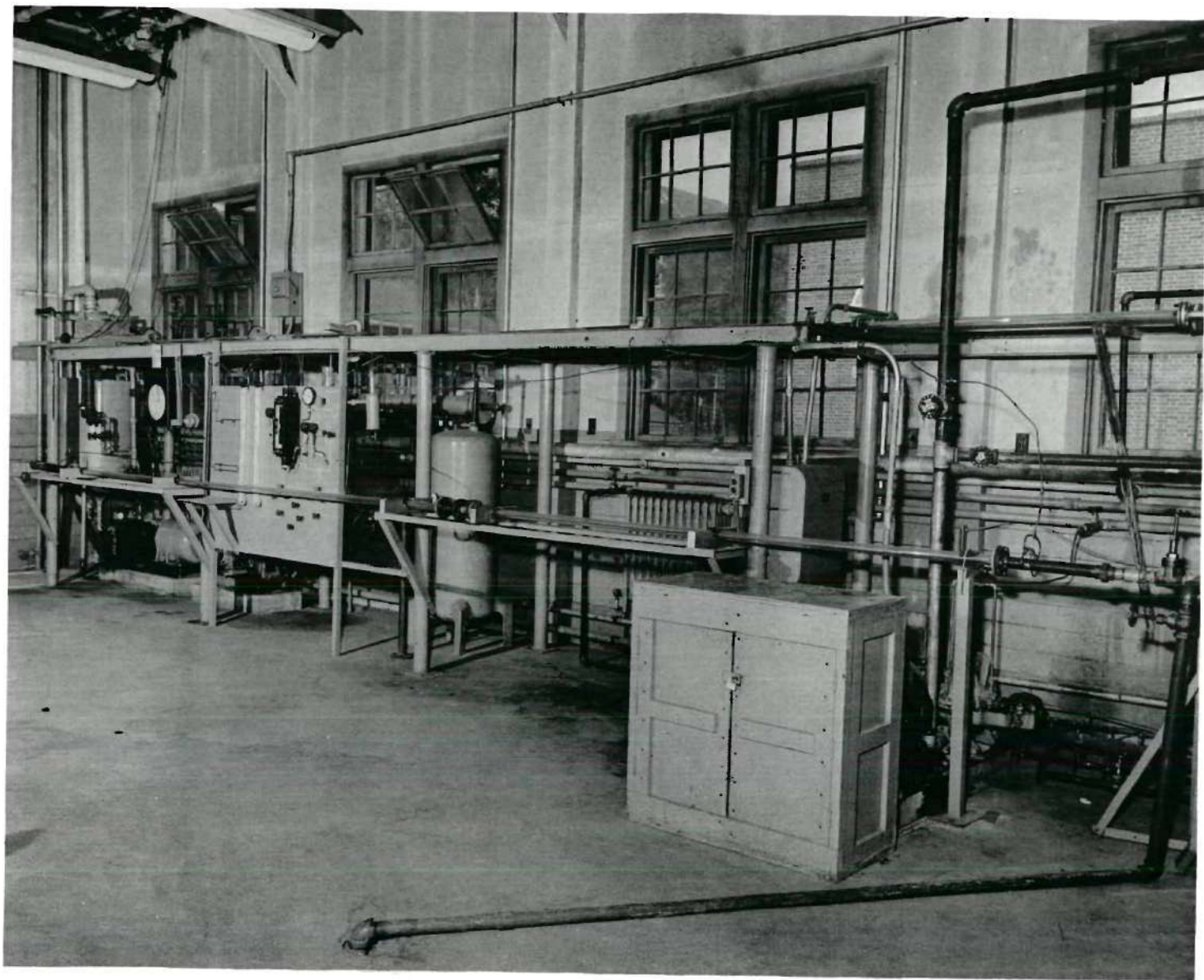


Figure 4. Experimental Apparatus.



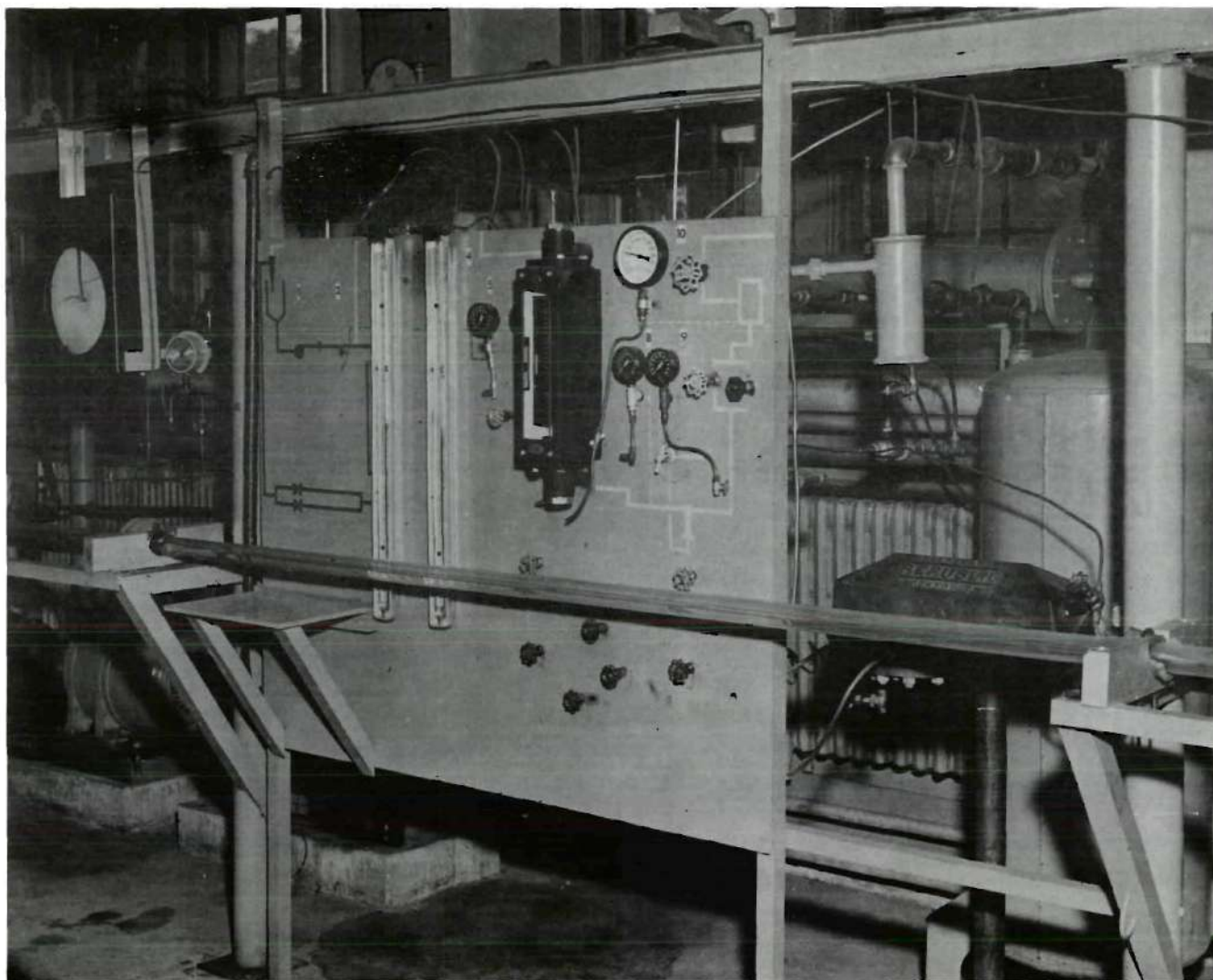


Figure 5. Control Panel.

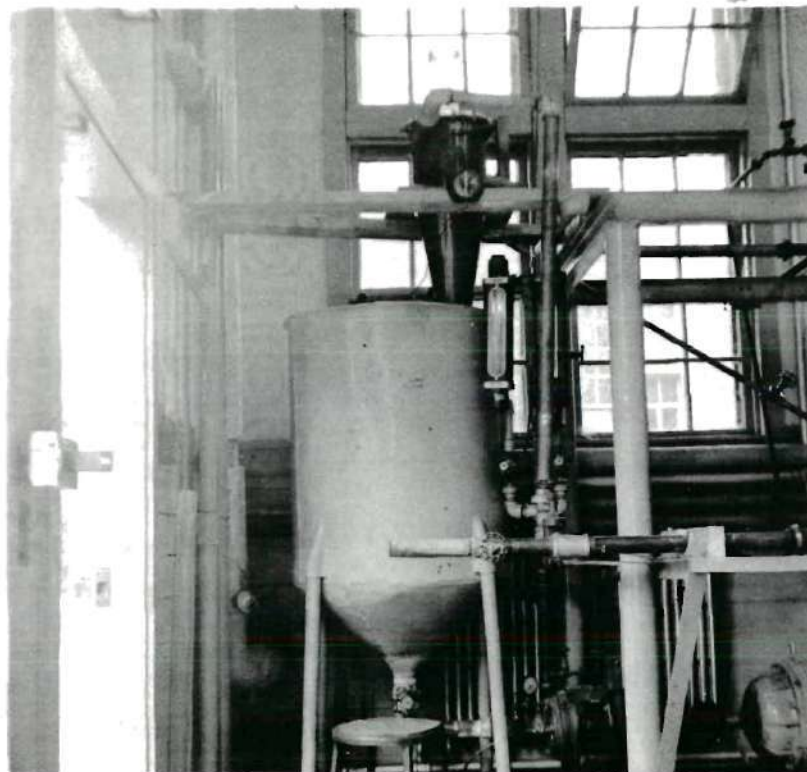


Figure 6. Water Storage with Pump and Discharge Line.

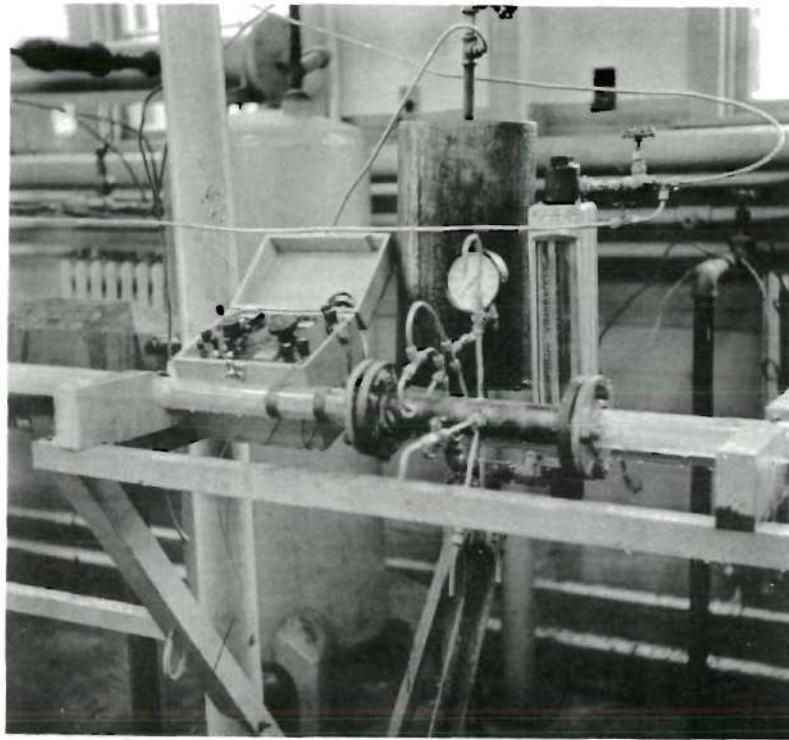


Figure 7. Test Section and Solute Injection System.

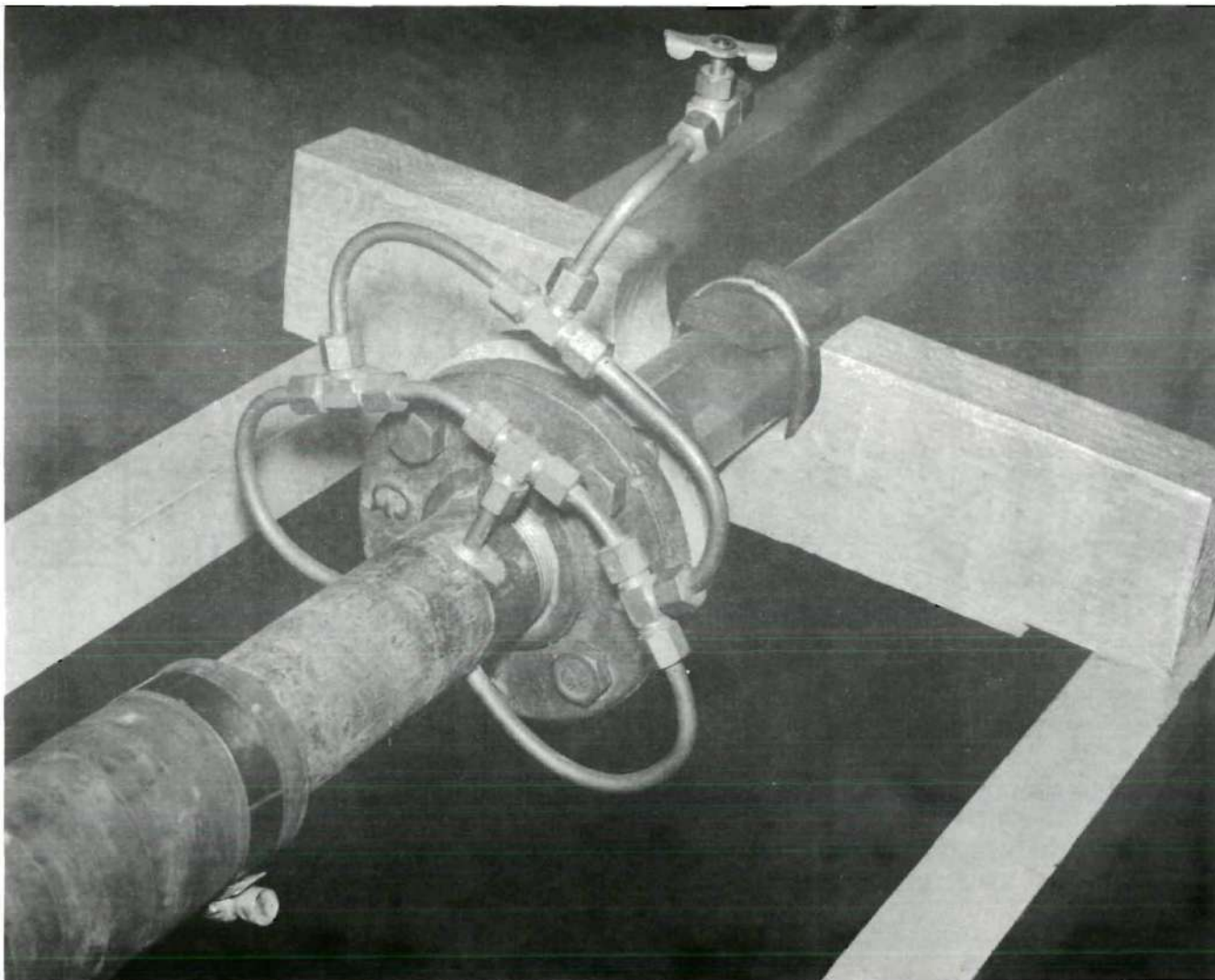


Figure 8. Pressure Pick-Up Device.



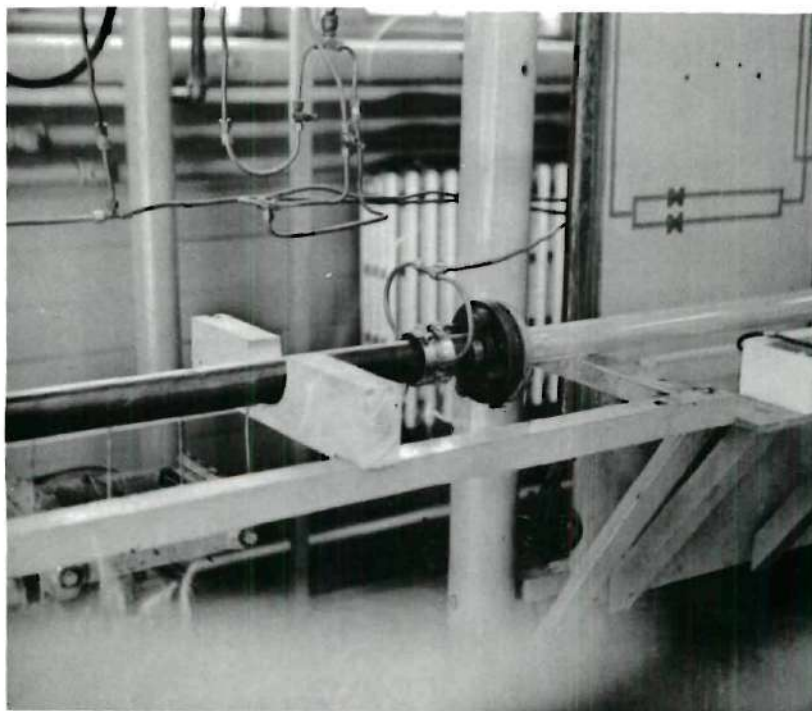


Figure 9. Mixing Length Section.



lowing paragraphs.

The Air Supply System.--Compressed air was furnished to an eight cubic foot storage tank by an Ingersoll-Rand 50 CFM compressor. The compressor discharge varied from about 90 to 105 psig. Two glass wool filters were placed in the air line, one immediately preceeding the storage tank, and one in the storage tank discharge line. The flow of air from the storage tank was regulated by two needle valves of different sizes which served as fine and coarse flow adjusters. After passing through the needle valves the air passed through a 3/8-inch Klipfel diaphragm-type reducing valve operated pneumatically. This valve reduced the pressure from the supply tank to any desired pressure between 0 and 60 psig. After throttling, the air flowed through a Schutte and Koerting model 18200 Safeguard Rotameter with an aluminum 61-K rotor. A calibration chart for the rotameter was furnished by the manufacturer. Pressure gauges, calibrated in this laboratory, were installed in the air line immediately preceeding and following the rotameter. The gauges were of the Bourdon type. The temperature of the air leaving the rotameter was measured by a mercury thermometer inserted in the air line. The air flowed from the rotameter into the test section through a 3/8-inch standard iron pipe which was teed at 90 degrees to the 1 1/2-inch water supply line. The end of the 3/8-inch pipe, after being brazed into the water pipe, was carefully rounded so that it would conform to the

interior surface of the water line. The air supply control panel is shown in Fig. 5.

The Water Supply System.--The water from the city supply was fed into a conical bottom storage tank having a capacity of approximately 125 gallons. The tank discharged to the intake of an Ingersoll-Rand 1 CORVNL pump which had a capacity of 75 GPM against a head of 120 feet of water. The water leaving the pump was metered through one of two small rotameters mounted in parallel or the rotameters were bypassed and an orifice was used to meter the flow. The smaller rotameter was a Fisher and Porter rotameter with a capacity of 2.5 GPM, the larger rotameter was a Schutte and Koerting rotameter with a capacity of 11.5 GPM. The orifice had an useful range of 14 to 50 GPM. A U-tube manometer containing mercury and water measured the pressure drop across the orifice. The three flow meters were calibrated in this laboratory. A recycle line from the pump discharge to the water supply tank was installed to allow the pump to operate at its rated capacity. Fig. 6 shows the storage tank, pump and rotameters.

The Mixing Section.--The mixing section consisted of 15 feet of 1 1/2-inch copper pipe flanged to a section of Pyrex Double Tough 1 1/2-inch glass pipe, 5 feet long, two Crane company number 1001 long-sweep 90-degree drainage elbows, and several additional feet of 1 1/2-inch schedule 80 iron pipe which served to connect the mixing section with the test section. This section should not be confused with the mixing

length section.

The Solute Injection System.--A cylindrical tank with a capacity of 2.4 gallons and tested to 105 psig served as a reservoir for the solute. An air line from the air supply tank to the solute supply tank furnished the necessary pressure. A Fisher Governor Company Type 67 pressure controller installed in the line maintained a constant pressure of 40 psig on the solution. The tank had a fill line and a drain line. The drain line was used to obtain samples of the solution for concentration determinations. The solution flowed from the tank through a Fisher and Porter rotameter having a capacity of 0.810 GPM. A copper-constantan thermocouple was placed in intimate contact with the line leading to the rotameter and insulated from the atmosphere. The potential of the thermocouple was measured with a Leeds and Northrup Serial 782642 potentiometer. The fluid passed from the rotameter through a 10-foot length of 1/4-inch copper tubing to a point 20 inches downstream of the orifice where the tubing teed into the 1 1/2-inch water line.

Lines were also provided from the rotameter to a point 4 inches upstream of the glass portion of the test section and to the injection ring on the mixing length section.

The Test Section.--The test section consisted of 194 inches of 1 1/2-inch Double Tough Pyrex Glass pipe for visual observation of the flow pattern and a section of 1 1/2-inch schedule 80 steel pipe was inserted in the center of the test



section. At this section of steel pipe the temperature and pressure were measured and means were provided whereby samples could be withdrawn from the flowing fluids (Fig. 7). The pressure was measured by a Bourdon-type pressure gauge calibrated in this laboratory and connected by 1/4-inch tubing to 3 points, 120 degrees apart, on the tube circumference. This "multiple pressure pick-up" device is shown in Fig. 8. The 1/4-inch tubing was inserted in a bored hole in the pipe and brazed in position, care being taken to insure a flush fit on the interior surface of the pipe wall. The steel section was connected to the glass pipe by flanges, the inside diameter of the flange conforming to the inside diameter of the pipe. A thermocouple lead was imbedded in the rubber flange gasket such that the bead protruded 1/8 inch into the stream. A Leeds and Northrup Serial 782642 potentiometer was used to measure the potential. At the center of the section, 4 sampling lines were located tapped at 0, 90, 135 and 180 degrees circumferentially from the top of the pipe. These 1/4-inch lines were brazed into the pipe such that the inner surface of the pipe remained smooth. Valves were fitted on each line to permit adjustment of the sampling rate. A 1/4-inch copper line was installed from the solute injection system to a point 4 inches upstream of the beginning of the glass section. This provided a means of injecting a dye into the stream.

The Mixing Length Section.--After the experimental determi-



nation of the  $V/L$  ratio by the concentration method was completed, a 5-foot length of schedule 80 steel pipe was installed in the test section replacing the last section of glass pipe. At the upstream end of this pipe a device similar to a piezometer ring allowed the solute to be injected into the stream. Sample take-off tubes were located every 6 inches downstream of the injection point for a distance of 54 inches. These  $1/8$ -inch copper tubes were set into the under side of the pipe and soldered into position such that they did not protrude past the inner wall of the pipe. The "piezometer" ring was constructed of 2 pieces of 2-inch I.D. radiator hose  $1/2$ -inch wide, two 2-inch hose clamps and a  $2 \times 8 \times 1/16$ -inch piece of galvanized tin. At the point of injection 17  $1/32$ -inch holes were drilled in the steel pipe, spaced  $1/8$  inch on center, the middle hole later positioned at the bottom of the horizontal pipe. The tin piece was fitted around the pipe forming an annulus, the hose acting as gaskets, and the device secured with the hose clamps. Two  $1/4$ -inch copper tubes were brazed into the tin and joined at a tee where the line from the solute system joined in. This section is shown in Fig. 9.

Solute injection points were provided 2 inches upstream of the glass test section and 3 inches upstream of a 1-inch glass pipe which ran parallel to the mixing section. This 1-inch line branched off from the  $1\ 1/2$ -inch mixing section 6 inches downstream of the point at which the air was

injected into the mixing section. The 1-inch line ran parallel with the mixing section for 22 feet and then rejoined the mixing section. This 1-inch pipe was all copper except for a 1-inch I.D. Pyrex Glass section 66 inches in length. These 2 injection points provided means whereby a dye could be injected into the stream and the mixing effects observed.

## CHAPTER IV

## EXPERIMENTAL PROCEDURE

The primary experimental purpose was to measure the V/L ratio by the concentration method and compare the results with the values obtained by metering the separate phases. The first step was the calibration of the flow meters and pressure gauges. The calibration of the air rotameter furnished by the manufacturer was accepted.

In determining the V/L ratio by the concentration method, the water rate was set at a constant value from approximately 0.7 to 50 GPM and the air rate varied from approximately 0.004 to 0.09 pounds per second. The solute was injected at a known rate and appropriate variables were measured. A typical run was made by setting the water rate at a steady value. The air rate was adjusted to some steady rate and the solute injection started. A previous injection of dye had shown how long it would take for the solute to reach the test section. After the proper length of time had elapsed, samples were taken from the test section and the solute supply tank. At the same time readings were taken of the measured variables and recorded. These measurements included the water rate, water temperature, air rate, air pressure, air temperature, solute rate, solute temperature, test section temperature and test section pressure. From



these data and the normalities of the samples, the V/L ratio could be computed. Then the air rate was changed and the procedure repeated except that samples were taken from the solute supply tank only at the start of a series of runs and at the end of a series of runs. Usually about 5 different air rates were used for each water rate.

In determining the two-phase mixing length, i.e., the length of pipe necessary for mixing when the solute was injected during two-phase flow, the procedure was about the same as described in the preceding paragraph except that the solute was injected through the "piezometer" ring on the mixing-length section. Samples were taken simultaneously from the sample take-off tubes and the same measurements were made except that the test section temperature and pressure were excluded.

The concentration of the solute, sodium hydroxide, was determined by titrating the samples with standard hydrochloric acid in the presence of phenolphthalein indicator.

Visual studies on the mixing lengths were made by injecting a solution of Rosaniline Hydrochloride (a red dye) or Malachite Green (a blue-green dye) at the injection points upstream of the glass observation sections. For these studies air and water rates were recorded and comments on the effects observed were recorded. The rate of injection of dye was not recorded since the injection rate was usually less than 5 per cent of the full scale reading.



## CHAPTER V

## DISCUSSION OF RESULTS

The experimental data and calculated results are presented in Tables 1, 2 and 3 and Figs. 10 and 11. The original data are on file in the School of Chemical Engineering of the Georgia Institute of Technology.

It should be noted that the experimental phases of this study necessarily contained some artificial factors. Since there was no mass transfer between the phases, the concentration of solute was the same at all points in the two-phase flow system, therefore it was not necessary to meter the solute into the stream. The only requirement was to maintain a steady flow of solute into the stream, then the concentrations  $C$  and  $C_0$  could be computed from one concentration measurement and the known mass flow rates. However, in order to simulate a system initially in single-phase flow, the sum of the mass flow rates of the air and water at the point of injection were considered to be the mass flow rate of a one-component system in single-phase flow and  $C_0$  was determined by metering the solute. This discussion will be based on the "premise" of single-phase flow existing initially.

The expression for the maximum error in the  $V/L$  ratio as derived in Appendix II is

$$\left[ \frac{d\left(\frac{V}{L}\right)}{\frac{V}{L}} \right]_{\max} = \left(1 + \frac{W_k}{W_g}\right) \left| \frac{dC}{C} \right| + \left(1 + \frac{W_k}{W_g}\right) \left| \frac{dC_o}{C_o} \right| + \left| \frac{dP}{P} \right| + \left| \frac{dT}{T} \right| \quad (23)$$

where P and T are measured at the point where the V/L ratio is desired. If one considers the V/L ratio determined by metering the phases as correct, then the error due to concentration technique may be expressed by e.

$$e = \frac{\left(\frac{V}{L}\right)_m - \left(\frac{V}{L}\right)_c}{\left(\frac{V}{L}\right)_m} \quad (24)$$

Substituting from equations (12) and (17) and remembering that  $v_g/v_l$  is determined by the same measurements for both methods one obtains

$$e = \frac{\frac{1}{v_l} \left(\frac{W_g}{W_l}\right)_m - \frac{v_g}{v_l} \left(\frac{C}{C_o} - 1\right)}{\frac{v_g}{v_l} \left(\frac{W_g}{W_l}\right)_m} = \frac{\left(\frac{W_g}{W_l}\right)_m - \left(\frac{C}{C_o} - 1\right)}{\left(\frac{W_g}{W_l}\right)_m} \quad (25)$$

Equation (25) gives an accurate estimate of the error in the V/L ratio by the concentration method provided that the error in the  $(W_g/W_l)_m$  ratio is small compared to e. Since  $(W_g/W_l)_m$  was determined by metering the phases separately, it was possible to assess the probable error in this value by estimating the uncertainty of each observation when the instruments were read.

Table 1 gives  $(W_g/W_l)_m$  and the estimated error in  $(W_g/W_l)_m$  due to instrument precision. The error, e, due to

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the concentration method is also given in Table 1.

Figure 10 is a graphic representation of the errors due to the concentration method. In presenting these data it was desired to separate the effect of pressure and temperature errors from errors due to the concentration measurements. Therefore the per cent error in the  $V/L$  ratio and equivalently in the  $W_g/W_l$  ratio was plotted against  $(W_g/W_l)_m$ . It can be seen from equation (16) that the concentration measurements alone determine the  $W_g/W_l$  ratio and that the temperature and pressure measurements at the test section are responsible for the ratio of the specific volumes  $v_g/v_l$ . Multiplication of  $(W_g/W_l)_m$  or  $(\frac{C}{C_o} - 1)$  by  $v_g/v_l$  gives the  $V/L$  ratio. Thus by equations (24) and (25)  $\bullet$  represents both the error in the  $(V/L)_c$  ratio and the  $(W_g/W_l)_c$  ratio. Therefore by using  $(W_g/W_l)_m$  as the abscissa the effects of errors from pressure and temperature measurements at the test section are eliminated.

Equation (23) also has been plotted in Figure 10 to show the maximum error that can be expected for one, two, and three per cent errors in the concentration measurements and no errors in the temperature and pressure measurements. Considering the values for  $W_g/W_l$  greater than 0.01 it was found that 63.5 per cent of the values indicated an accuracy of 1 per cent or better in concentration measurements, 79.3 per cent showed an accuracy of 2 per cent or better, and 92.1 per cent indicated an accuracy of 3 per cent or better.



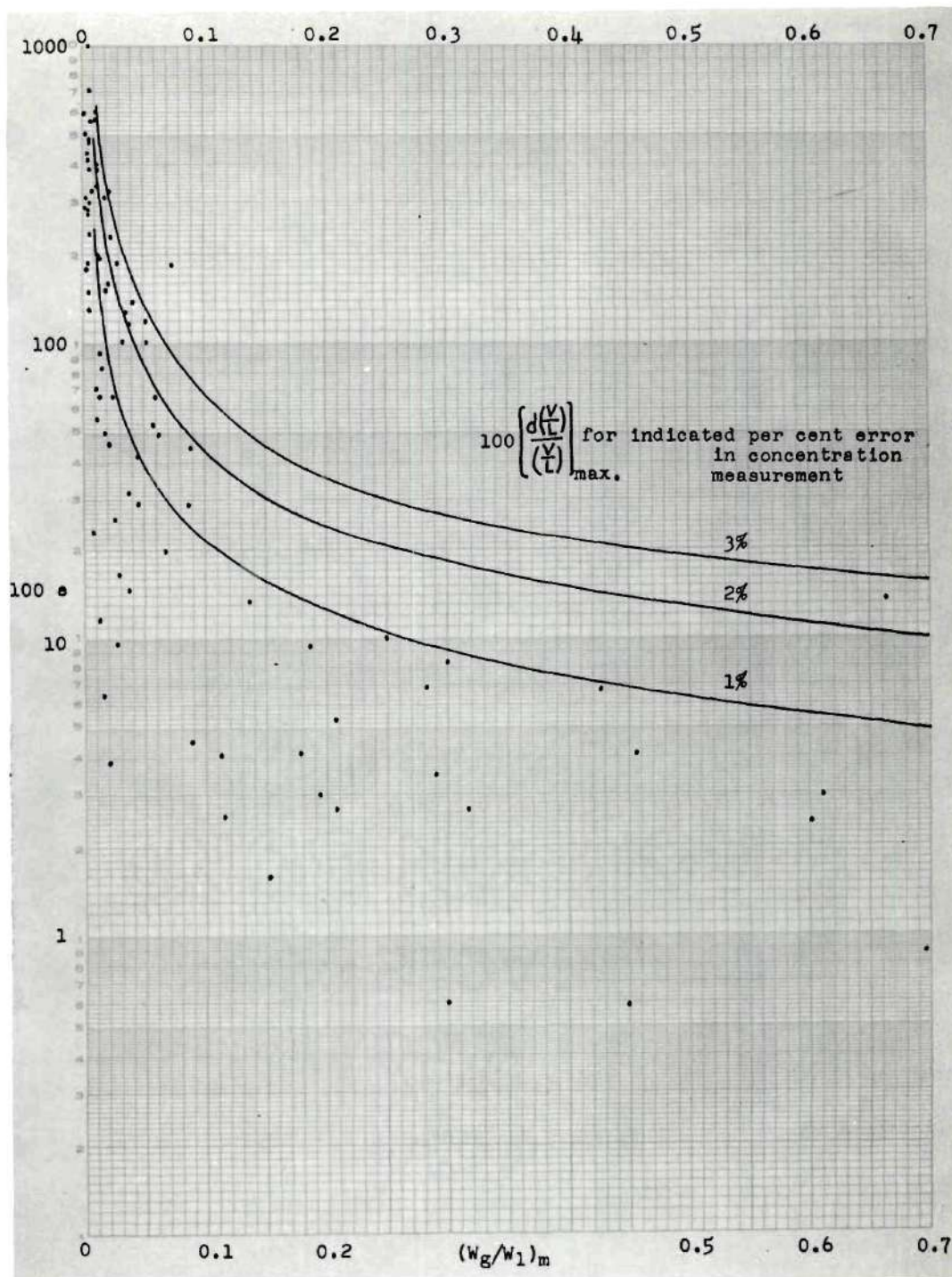


Figure 10. Per Cent Error in  $V/L$  Ratio (or  $W_g/W_l$  Ratio) Measured by Concentration Method Versus  $(W_g/W_l)_m$ .

A major source of error was the length of time necessary for the fluid to flow from the point of injection to the test section, i.e., steady state could not be maintained. In some cases this was as long as two and one-half minutes. Because the flow rates sometimes tended to change slightly with time due to varying back pressure on the water pump and solute tank, it was occasionally difficult to obtain the readings for the water rate and solute rate that corresponded to the sample. This hold up was often the greatest factor in estimating the uncertainty in the flow rates. In analyzing the withdrawn samples, the normality determined by the titration was accurate to 1 per cent, but the uncertainty in metering the mass flow rates increased the error in  $C_0$  considerably. If  $C_0$  had been determined by withdrawing a sample from the stream during single-phase flow, then the  $V/L$  ratio determined by the concentration method would be independent of the mass flow rate. Obviously this was not appropriate in this investigation. The range of applicability of the concentration method on a particular system is largely determined by the accuracy to which the concentration may be determined.

Up to this point the ratio of the specific volumes has been dealt with only briefly. In most two-phase systems temperature measurements present no problem but the pressure determination in two-phase flow often causes considerable concern. In flow regimes such as stratified or annular the



pressure at a particular point remains constant with time whereas in slug or plug flow, the pressure varies greatly. Rapid fluctuations between 5 and 20 psig were observed during plug flow. In a case like this, the V/L ratio was somewhat of an average value and although the limits of the ratio were established the variation of 100 per cent in the pressure makes the V/L ratio somewhat vague. So far this is an inherent difficulty in characterizing two-phase flow.

The use of sodium hydroxide as the solute was very satisfactory. Determination of the normalities of the samples by titration was accurate but time-consuming. Although a large proportion of the pipe network was mild steel, no increase in corrosion was noted over a two-month period. In fact, the presence of an 8 per cent NaOH solution in the supply tank arrested the formation of rust in the tank.

During the earlier part of the experimental program the solute was injected at a point where the two-phase flow pattern had developed, about 35 feet upstream of the test section. Samples were taken simultaneously from the lower 3 sampling tubes at the test section but it was found that the concentration of the different samples varied as much as 10 per cent for the same run. The injection point was then repositioned to a point 20 inches downstream of the orifice such that the solute was injected during the single-phase flow of water. The concentration of the samples agreed when the solute was injected at the latter point.

Similar inconsistencies were observed later in the investigation when an attempt was made to measure the two-phase mixing length. The solute was injected through the "piezometer" ring in the mixing length section and samples were taken every 4 diameters down the pipe for a distance of five feet. Since the concentration could be determined only at the pipe wall the length necessary for mixing could be found by noting the point at which the concentration leveled off. However, in checking the consistency of the data obtained, it was found that even though the concentration was constant after a certain point it was not possible to obtain a mass balance on the sodium hydroxide using the known flow rates and the measured concentration. Only when the flow was annular or stratified could a mass balance be obtained. In order to gain an insight into this problem, the solute was changed and a dye used. This dye was injected at a point several inches upstream of the glass observation section. Upon injecting the dye into the two-phase stream it became obvious why the concentration measurements were inconsistent in the first attempt at measuring the V/L ratio and in the mixing-length determinations. When the dye was injected at a steady rate and the water rate at the section was fluctuating due to waves or slugs, the dye was not evenly distributed in the fluid. In slug flow the effect was most noticeable. By observing a point several feet downstream of the injection point it was noticed that prior to a slug



flowing past the section the fluid appeared to be a uniform color but as the slug passed the section the color was intensified. The fluid immediately behind the slug was almost clear but as more fluid flowed past, the color deepened until another slug passed and the color was again suddenly intensified. Although just the bulk of the fluid in the slug would tend to give the impression of a darker color, this was not the case. The concentration definitely began to build up as indicated by the color in the fluid preceeding the slug and increased suddenly in the slug. This effect was verified by three observers. Depending on the type flow this effect became less noticeable down the length of the pipe but the effect was observed after the fluid had flowed through 40 feet of pipe, an open gate valve, and a sudden expansion. A series of runs was made at water rates from 0.7 to 50 GPM and at air rates from 0 to 0.09 pounds per second. Ten water rates were used and the air rate varied continuously from no flow to maximum flow at each water rate.

On the basis of these runs it appeared that only during smooth flow, i.e., annular, stratified or single-phase, did the mixing become complete within twenty feet. For these types of flows, the dye was injected proportional to the fluid flowing past the point of injection. As irregularities (waves, slugs, plugs) appeared in the flow the average concentration with respect to time at a cross-section was not constant. Because the waves or slugs moved at a

velocity greater than the average velocity of the water, the effect was mitigated down the length of the pipe but the effect often was still noticeable at the last point of observation, approximately forty feet from the point of injection. As the frequency of the irregularities increased, the effect was mitigated.

This seemed to explain the lack of agreement between the concentration of the samples when the samples were taken at different points on the periphery of the tube. Samples taken at the point 90 degrees from the top contained fluid mainly from the slugs while the sample taken from the bottom of the tube had a much lower concentration. In measuring the mixing length, this also explained why the samples often analyzed at a much lower concentration than was indicated by a mass balance.

The conclusion to be drawn from this is that unreliable results may be obtained if the solute is injected during two-phase flow.

The method of sampling, withdrawing fluid from the pipe at the surface, presented no problems but then it must be kept in mind that these samples were gross samples in that they represented the fluid flowing past a section for as much as ten seconds. The criteria was that the sampling rate should not exceed 2 per cent of the total water rate or 5 cubic centimeters per second, whichever was less. At the lowest flow rate of 0.7 GPM this required a sampling ratio of

one cubic centimeter per second and ten seconds to obtain the required sample. In taking samples, two outlets were used, the tubes 135 and 180 degrees circumferentially from the top of the pipe. If the concentration of the samples obtained agreed within 1 1/2 per cent, the run was judged acceptable. An initial study showed that the concentration of samples taken simultaneously from all four tubes agreed within the limits of the accuracy of titration. It may be noted that only in single-phase flow was the top sampling tube used.

As mentioned before, an attempt was made to measure the two-phase mixing length, but the visual observations indicated that the mixing lengths were longer than could be measured on this apparatus. However, it was felt that an indication of the single-phase mixing length would be in order as a guide for utilization of the concentration method. In order to have found more than an order of magnitude approximation, it would have been necessary to consider solute types, methods of injection, state of velocity profile development and perhaps other variables. Accordingly, a scheme was developed to visually observe the mixing length when a dye was injected at a single point at the pipe wall. The point of mixing was that point downstream of the injection point at which no color gradients could be observed. Since this observation was subjective, three different observers made independent observations. The results are presented in Fig. 11 and Tables 3 A - D. The data in Tables 3A and 3B



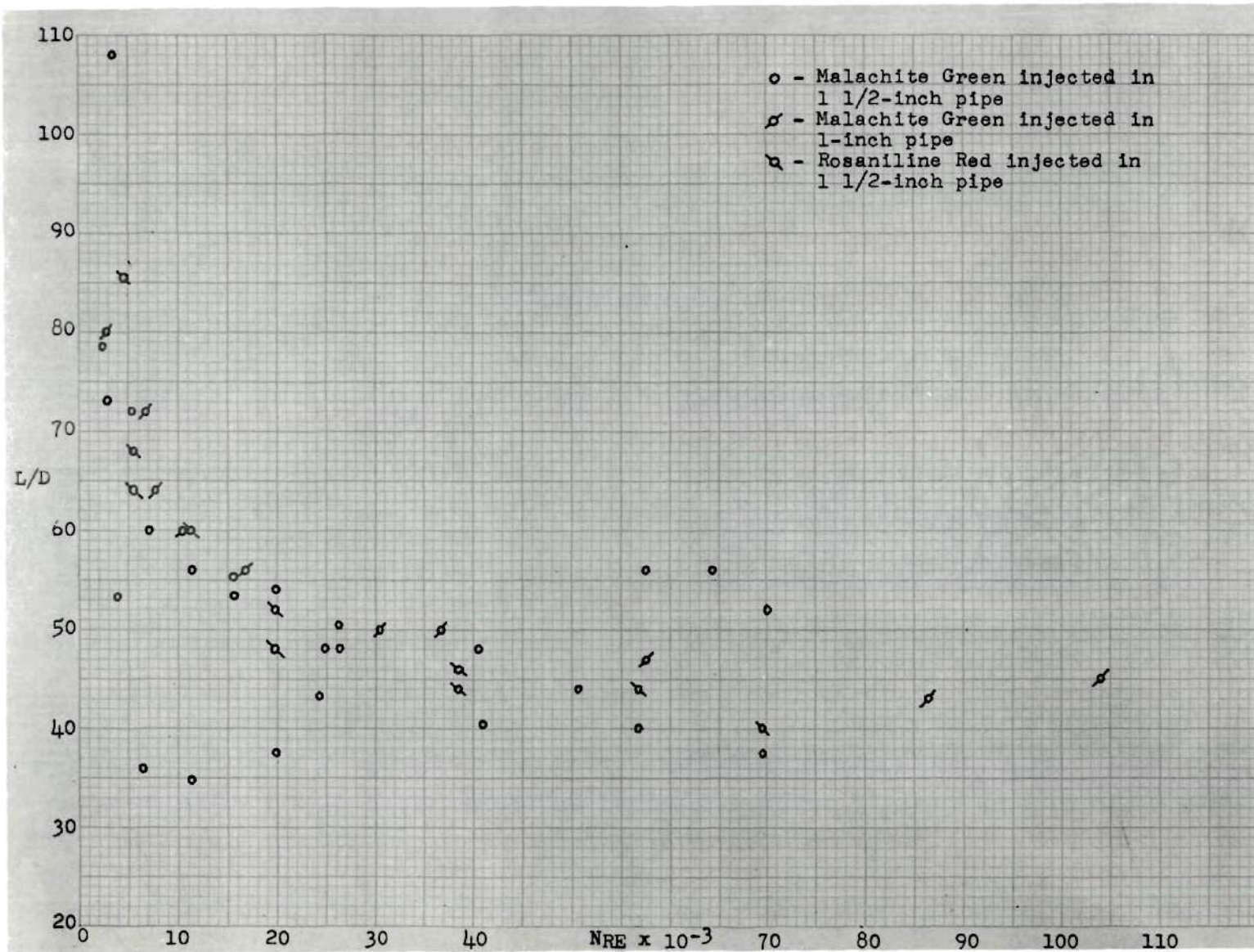


Figure 11. Single-Phase Mixing Length Versus Reynolds Number for 1-Inch and 1 1/2-Inch Glass Pipe.



are the results of one observer, and Tables 3C and 3D are the observations of the other two observers. In all the observations the degree of uncertainty was estimated at approximately eight diameters. Because of the limited observations, no conclusions can be drawn. However, because the injection rate was less than 0.04 GPM and the pipe was smooth, these data should furnish the basis for estimating the maximum mixing length that may be expected under similar conditions.

## CHAPTER VI

## CONCLUSIONS

The conclusions resulting from the evaluation of the concentration method for determining the  $V/L$  ratio in an air-water system may be stated in summary as follows:

1. The concentration method may be used to determine the value of the  $V/L$  ratio in air-water systems with the accuracy decreasing as the  $W_g/W_l$  ratio decreases.

2. The solute should be injected during single-phase flow since it cannot ordinarily be injected into a two-phase flow system such that it is evenly distributed in one phase.

## CHAPTER VII

### RECOMMENDATIONS

On the basis of this investigation, it is suggested that additional study can be profitably pursued on other systems, specifically the steam-water system. It appears that good results can be obtained at all but the lowest qualities but additional development is needed on a system with mass transfer between the phases. Should an investigation in this line be pursued, the following recommendation is offered to improve the operation of the concentration method.

Although the use of air pressure provided a satisfactory means of injecting the solute, a slight improvement may be made by the use of a gear pump or some similar device that has an output independent of the discharge pressure. The only requirement for the solute injection is that the rate of injection be constant. The actual rate is not needed since better results will be obtained by the measurement of  $C_0$  rather than computing  $C_0$  by a material balance.

## APPENDICES



## APPENDIX I

DATA AND RESULTS OF ALL RUNS

Table 1. Results of V/L Determination by the Concentration Method

Run No.	$W_g$ (lb/sec)	$(W_g/W_l)_m$	estimated % error in $(W_g/W_l)_m$	$N_{RE_1}$	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
101 *	0.0777	0.299	2.94	4,400	252.5	273.6	8.34
102	0.0772	0.299	2.78	4,377	253.5	252.0	- 0.60
103	0.0500	0.207	3.24	4,182	175.8	166.4	- 5.34
104	0.0424	0.178	3.37	4,145	151.6	158.0	4.16
105	0.0292	0.112	3.69	4,594	95.0	98.9	4.09
107	0.0405	0.152	3.33	4,689	129.5	131.5	1.57
110	0.00969	0.0363	5.98	4,704	30.9	40.7	31.6
111	0.0874	0.326	2.80	4,681	277.3	269.6	- 2.78
112	0.0812	0.290	2.73	4,935	246.8	238.3	- 3.46
113	0.0914	0.207	2.76	7,799	176.0	180.6	2.63
114	0.0633	0.138	2.94	8,070	116.7	132.4	13.4
115	0.0413	0.0898	3.32	8,074	76.2	110.1	44.5

\* Note: Runs 1 through 87 were for calibration and familiarization and are not reported. Runs 88-100 were not made.

Table 1. Results of V/L Determination by the Concentration Method (con't.)

Run No.	$\frac{W_g}{(lb/sec)}$	$(W_g/W_l)_m$	estimated % error in $(W_g/W_l)_m$	$N_{RE_l}$	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
116	0.0287	0.0609	3.84	8,118	51.5	77.2	49.9
117	0.0190	0.0407	4.50	8,128	34.5	82.6	139.6
118	0.0103	0.0216	5.80	8,325	18.3	78.1	326.5
119	0.0903	0.1244	4.01	12,333	105.2	9,076	8,531.0
120	0.0645	0.0865	2.46	12,455	72.9	93.9	28.9
121	0.0442	0.0598	2.85	12,522	50.4	77.6	53.8
122	0.0186	0.0247	3.99	12,785	20.8	34.6	66.5
123	0.00997	0.0134	5.45	12,598	10.6	17.6	66.6
124	0.0902	0.0886	3.83	16,997	69.9	66.7	-4.55
125	0.0680	0.0660	3.71	17,207	52.0	41.6	-19.9
126	0.0454	0.0440	2.41	17,239	37.0	47.8	29.0
127	0.0277	0.0269	2.97	17,240	22.7	20.5	- 9.67
128	0.0112	0.0108	4.64	17,403	9.06	3.96	-56.3
129	0.0945	0.0744	1.77	21,882	58.9	167.7	184.6
130	0.0553	0.0515	3.77	21,876	40.8	82.1	101.3

Table 1. Results of V/L Determination by the Concentration Method (con't.)

Run No.	$\frac{W_g}{(lb/sec)}$	$(W_g/W_l)_m$	estimated % error in $(W_g/W_l)_m$	$N_{RE_l}$	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
131	0.0450	0.0349	5.63	22,047	28.5	65.1	128.5
132	0.0313	0.0240	5.69	22,010	19.8	66.9	238
133	0.0116	0.00886	10.70	22,084	7.47	49.4	562
134	0.00692	0.00529	6.68	21,955	4.46	53.1	1,092
135	0.00559	0.00351	7.77	26,405	2.95	40.6	1,276
136	0.0161	0.0101	3.94	26,331	8.52	56.6	565
137	0.0306	0.0195	5.54	25,984	16.4	67.4	310
138	0.0456	0.0290	4.43	25,996	24.4	70.1	187
139	0.0927	0.0602	2.64	25,334	47.4	79.1	66.8
140	0.0594	0.0379	3.22	25,853	30.4	65.9	116.6
141	0.0104	0.00375	9.90	46,705	3.09	16.90	446.0
142	0.0102	0.00370	10.04	46,686	3.05	8.74	187.0
143	0.0207	0.00768	6.38	44,115	6.23	-18.3	-394.0
144	0.0333	0.0125	5.08	42,480	9.80	8.65	- 11.7
145	0.0440	0.0167	4.65	42,017	12.3	2.11	- 82.9



Table 1. Results of V/L Determination by the Concentration Method (con't.)

Run No.	$\frac{W_g}{(lb/sec)}$	$(\frac{W_g}{W_l})_m$	estimated % error in $(\frac{W_g}{W_l})_m$	$N_{RE1}$	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
146	0.0670	0.0263	3.86	40,688	18.3	13.6	- 25.4
147	0.0813	0.0324	3.6	40,058	20.6	- 0.46	-102.2
148	0.00933	0.00251	10.6	59,438	2.00	-26.6	-1,428.0
149	0.0178	0.00485	6.8	59,089	3.87	14.4	272.0
150	0.0317	0.00865	4.9	59,052	6.18	-14.0	- 326.0
151	0.0472	0.0131	4.0	56,931	9.02	26.5	193.0
152	0.0638	0.0182	3.4	55,569	10.8	-16.1	-1,584.0
153	0.0744	0.0198	5.5	59,589	11.2	- 6.85	- 161.0
154	0.00385	0.000601	14.1	101,495	.43	-17.5	-4,184.0
155	0.0175	0.00283	7.4	95,394	1.85	12.78	590.0
156	0.0358	0.00569	3.1	97,026	3.20	- 6.41	- 299.0
157	0.0272	0.00424	3.5	99,107	2.51	- 7.83	- 412.0
158	0.0212	0.00326	4.0	100,530	2.02	- 23.1	-1,239.
159	0.0101	0.00183	6.3	85,589	1.34	5.53	312.
160	0.0185	0.00332	4.3	86,243	2.30	- 1.81	- 179.

Table 1. Results of V/L Determination by the Concentration Method (con't.)

Run No.	$\frac{W_g}{(lb/sec)}$	$(W_g/W_l)_m$	estimated % error in $(W_g/W_l)_m$	NRE <sub>1</sub>	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
161	0.0291	0.00519	3.4	86,663	3.22	7.43	- 130.
162	0.0397	0.00716	3.0	85,591	4.23	-25.6	- 704.
163	0.0500	0.00908	3.2	85,104	5.12	3.93	- 23.3
164	0.0104	0.00232	6.4	69,335	1.81	7.03	288.
165	0.0200	0.00439	4.1	70,353	3.21	12.4	285.
166	0.0349	0.00778	3.2	68,797	5.09	29.0	470.
167	0.0515	0.0112	2.7	70,561	6.95	-17.0	- 344.
168	0.0667	0.0146	2.5	69,855	8.64	0.54	- 93.7
169	0.0828	0.0183	2.6	69,208	9.88	4.91	- 50.3
170	0.0101	0.00268	6.1	57,462	2.18	-32.6	-1,589.
171	0.0193	0.00513	4.2	57,525	4.08	- 2.05	- 150.
172	0.0219	0.00583	4.0	57,452	4.35	14.6	235.
173	0.0395	0.0105	3.0	57,618	7.37	2.18	- 70.4
174	0.0562	0.0149	3.6	57,538	9.77	9.13	- 6.44
175	0.0776	0.0202	2.4	58,818	11.9	11.5	- 3.88

Table 1. Results of V/L Determination by the Concentration Method (con't.)

Run No.	$W_g$ (lb/sec)	$(W_g/W_l)_m$	estimated % error in $(W_g/W_l)_m$	$N_{RE_l}$	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
176	0.0796	0.0209	2.6	58,142	11.8	6.42	- 45.6
177	0.0742	0.664	3.3	1,734	558.0	483.0	- 13.5
178	0.0798	0.697	4.3	1,791	588.0	593.0	0.88
179	0.0522	0.457	5.8	1,823	386.0	402.0	4.11
180	0.0346	0.316	5.0	1,847	260.0	274.0	2.74
181	0.0214	0.186	5.8	1,869	158.0	173.0	9.4
182	0.00706	0.00369	11.3	33,648	3.13	18.9	503.0
183	0.00453	0.00238	16.4	31,448	2.00	24.8	1,140.0
184	0.0138	0.00732	7.0	30,426	6.13	35.2	474.0
185	0.0230	0.0121	5.2	30,034	10.1	48.1	376.0
186	0.0395	0.0211	3.9	29,625	17.6	45.6	159.0
187	0.0559	0.0297	3.2	29,664	23.7	27.5	16.4
188	0.0701	0.0370	3.9	29,884	28.4	24.2	- 14.7
189	0.0836	0.0437	3.7	30,115	31.2	44.1	41.3
190	0.0127	0.114	5.7	1,730	95.9	93.5	- 2.52

Table 1. Results of V/L Determination by the Concentration Method (concluded)

Run No.	$W_g$ (lb/sec)	$(W_g/W_1)_m$	estimated % error in $(W_g/W_1)_m$	$N_{RE1}$	$(V/L)_m$	$(V/L)_c$	100 e, % error in $(V/L)_c$
191	0.0213	0.194	4.7	1,754	164.0	159.0	- 3.00
192	0.0319	0.282	3.9	1,835	239.0	222.0	- 6.9
193	0.0504	0.449	3.3	1,816	381.0	383.0	0.59
194	0.0684	0.601	3.1	1,852	510.0	497.0	- 2.40
195	0.0673	0.611	3.6	1,821	351.0	341.0	- 2.93



Table 2. Data for the V/L Ratio Determination by the Concentration Method

Run No.	E <sub>s</sub>	E <sub>t</sub>	h <sub>s</sub>	Q <sub>w</sub>	Q <sub>g</sub>	P <sub>g</sub>	T <sub>g</sub>	N <sub>s</sub>	N <sub>t</sub>	P <sub>t</sub>
101	1.16	1.02	13.2	1.78	41.8	33.5	302.2	1.857	0.0926	14.7
102	1.16	1.03	15.0	1.75	41.6	33.4	302.0	1.857	0.1045	14.7
103	1.16	1.07	13.9	1.64	30.4	26.2	302.0	1.857	0.1023	14.7
104	1.16	1.08	14.5	1.61	28.5	21.5	302.0	1.857	0.1101	14.7
105	1.16	1.09	15.6	1.77	21.6	17.7	302.1	1.857	0.1080	14.7
107	1.16	1.10	15.2	1.81	27.5	21.1	302.7	1.857	0.1028	14.7
110	1.16	1.10	14.4	1.82	7.8	15.0	302.7	1.857	0.0979	14.7
111	1.16	1.08	13.9	1.83	46.0	35.0	302.0	1.857	0.0924	14.7
112	1.16	1.10	15.0	1.91	41.8	36.5	301.0	1.857	0.0955	14.7
113	1.17	1.09	14.0	3.31	46.0	38.2	301.2	1.857	0.0571	14.7
114	1.17	1.08	15.0	3.21	36.5	29.0	300.4	1.857	0.0594	14.7
115	1.17	1.08	15.2	3.21	27.9	21.2	300.7	1.857	0.0614	14.7
116	1.39	1.04	13.9	3.30	21.2	17.7	300.2	1.760	0.0517	14.7
117	1.38	1.06	13.3	3.27	15.0	15.5	300.8	1.760	0.0512	14.7
118	1.38	1.06	14.1	3.35	8.3	15.0	301.0	1.760	0.0538	14.7

Table 2. Data for the V/L Ratio Determination by the Concentration Method (con't.)

Run No.	E <sub>s</sub>	E <sub>t</sub>	h <sub>s</sub>	Q <sub>w</sub>	Q <sub>g</sub>	P <sub>g</sub>	T <sub>g</sub>	N <sub>s</sub>	N <sub>t</sub>	P <sub>t</sub>
119	1.37	1.02	29.9	5.22	45.8	37.5	300.2	1.760	0.0715	14.7
120	1.37	0.99	21.4	5.22	36.5	30.0	299.2	1.760	0.0511	14.7
121	1.37	1.01	19.8	5.19	28.3	23.5	300.0	1.760	0.0478	14.7
122	1.35	1.01	20.1	5.30	14.7	15.5	301.0	1.760	0.0468	14.7
123	1.36	1.01	20.1	5.22	8.0	15.0	300.8	1.760	0.0472	15.7
124	1.35	0.99	32.5	7.10	44.5	39.6	300.1	1.760	0.0567	15.7
125	1.35	0.99	31.4	7.20	37.0	32.4	299.2	1.760	0.0535	15.7
126	1.35	0.99	28.5	7.23	28.3	24.7	299.4	1.760	0.0494	14.7
127	1.33	0.99	28.5	7.23	19.5	19.5	300.0	1.760	0.0487	14.7
128	1.33	0.99	33.2	7.27	8.9	15.3	300.1	1.760	0.0565	14.7
129	1.22	1.05	30.3	8.94	46.0	41.1	303.4	1.765	0.0478	15.7
130	1.19	1.03	28.2	9.07	36.5	32.0	302.2	2.220	0.0502	15.7
131	1.20	1.03	22.1	9.15	27.6	25.8	302.2	2.220	0.0384	15.2
132	1.20	1.01	22.0	9.24	21.0	21.5	302.2	2.220	0.0383	15.0
133	1.20	1.01	23.6	9.28	8.9	16.5	302.2	2.220	0.0408	14.7

Table 2. Data for the V/L Ratio Determination by the Concentration Method (con't.)

Run No.	E <sub>s</sub>	E <sub>t</sub>	h <sub>s</sub>	Q <sub>w</sub>	Q <sub>g</sub>	P <sub>g</sub>	T <sub>g</sub>	N <sub>s</sub>	N <sub>t</sub>	P <sub>t</sub>
134	1.20	1.00	21.9	9.28	5.4	16.0	303.2	2.220	0.0381	14.7
135	1.20	0.98	22.3	11.33	4.3	16.5	304.2	2.220	0.0315	14.7
136	1.20	0.98	24.8	11.29	11.7	18.5	304.2	2.220	0.0357	14.7
137	1.20	0.98	23.1	11.14	20.3	22.2	304.2	2.220	0.0336	14.7
138	1.20	0.98	25.3	11.14	27.2	27.4	304.2	2.220	0.0368	14.7
139	1.20	0.97	27.0	10.91	44.2	43.0	304.2	2.220	0.0396	15.7
140	1.21	0.97	23.7	11.14	33.0	31.5	302.2	2.220	0.0340	15.4
141	1.22	1.00	24.1	19.8	7.0	21.5	304.2	2.220	0.0191	15.0
142	1.22	1.00	23.1	19.8	6.9	21.5	304.7	2.220	0.0181	15.0
143	1.22	0.96	28.8	19.2	13.0	24.7	304.7	2.220	0.0227	15.2
144	1.22	0.92	27.1	19.0	19.5	28.5	304.7	2.220	0.0221	15.7
145	1.22	0.92	23.5	18.8	24.5	31.5	304.7	2.220	0.0190	16.7
146	1.22	0.92	28.2	18.2	33.5	39.1	304.2	2.220	0.0240	17.7
147	1.22	0.92	30.0	17.9	38.7	43.0	303.2	2.220	0.0253	19.3
148	1.23	0.92	31.9	26.6	5.8	25.2	303.2	2.220	0.0182	15.4



Table 2. Data for the V/L Ratio Determination by the Concentration Method (con't.)

Run No.	E <sub>s</sub>	E <sub>t</sub>	h <sub>s</sub>	Q <sub>w</sub>	Q <sub>g</sub>	P <sub>g</sub>	T <sub>g</sub>	N <sub>s</sub>	N <sub>t</sub>	P <sub>t</sub>
149	1.23	0.93	28.7	26.3	10.5	28.0	303.2	2.220	0.0173	15.4
150	1.22	0.92	26.7	26.3	17.4	32.5	304.2	2.220	0.01535	17.2
151	1.22	0.91	26.8	25.7	24.2	37.1	304.2	2.220	0.01660	17.9
152	1.22	0.91	36.4	25.0	30.6	42.5	304.2	2.220	0.01652	20.7
153	1.22	0.91	27.9	26.9	33.6	47.9	304.2	2.220	0.01569	21.7
154	1.20	0.90	29.5	46.1	2.0	36.5	305.5	1.592	0.00720	17.2
155	1.19	0.86	24.7	44.4	8.6	40.5	305.2	1.592	0.00646	18.7
156	1.19	0.86	20.1	45.2	16.0	48.9	304.9	1.592	0.00495	21.7
157	1.19	0.86	23.3	46.1	12.4	46.9	304.2	1.592	0.00386	20.7
158	1.19	0.86	24.8	46.8	9.9	44.9	304.0	1.592	0.00581	19.7
159	1.19	0.86	26.8	39.8	5.4	34.5	304.2	1.592	0.00777	16.7
160	1.19	0.86	27.5	40.1	9.4	38.0	304.2	1.592	0.00784	17.7
161	1.19	0.86	27.0	40.3	13.9	42.9	304.2	1.592	0.00775	19.7
162	1.19	0.86	26.9	39.8	18.2	46.4	304.2	1.592	0.00737	20.7
163	1.19	0.86	23.5	39.6	22.0	50.4	303.7	1.592	0.00675	21.7



Table 2. Data for the V/L Ratio Determination by the Concentration Method (con't.)

Run No.	E <sub>s</sub>	E <sub>t</sub>	h <sub>s</sub>	Q <sub>w</sub>	Q <sub>g</sub>	P <sub>g</sub>	T <sub>g</sub>	N <sub>s</sub>	N <sub>t</sub>	P <sub>t</sub>
164	1.19	0.86	31.1	32.2	6.0	29.5	303.7	1.592	0.01123	15.7
165	1.19	0.86	28.2	32.7	10.8	33.5	303.7	1.592	0.01002	16.7
166	1.20	0.85	28.4	32.2	17.6	38.5	304.2	1.592	0.0107	18.7
167	1.20	0.85	28.1	33.0	24.2	44.3	304.2	1.592	0.00942	19.7
168	1.20	0.85	28.2	32.7	29.5	49.9	304.2	1.592	0.00979	20.7
169	1.20	0.85	23.6	32.4	34.5	56.2	304.2	1.592	0.00813	22.7
170	1.20	0.85	26.7	26.9	6.2	25.8	304.2	1.592	0.0109	15.0
171	1.20	0.85	30.3	26.9	11.1	29.5	304.2	1.592	0.0129	15.4
172	1.20	0.85	26.1	26.9	12.6	29.5	304.2	1.592	0.01123	16.4
173	1.20	0.85	35.5	26.9	20.7	35.5	304.2	1.592	0.0167	17.4
174	1.20	0.85	34.3	26.9	27.6	40.5	304.2	1.592	0.0162	18.7
175	1.20	0.85	34.8	27.5	34.5	49.4	304.2	1.592	0.0161	20.7
176	1.20	0.85	32.5	27.2	35.2	49.9	304.2	1.592	0.0150	21.7
177	1.20	0.91	11.7	0.72	40.9	32.0	302.7	1.592	0.168	14.7
178	1.20	0.93	13.0	0.73	43.0	33.5	302.7	1.592	0.194	14.7
179	1.20	0.96	12.8	0.73	32.8	24.7	303.2	1.592	0.193	14.7

Table 2. Data for the V/L Ratio Determination by the Concentration Method (con't.)

Run No.	E <sub>s</sub>	E <sub>t</sub>	h <sub>s</sub>	Q <sub>w</sub>	Q <sub>g</sub>	P <sub>g</sub>	T <sub>g</sub>	N <sub>s</sub>	N <sub>t</sub>	P <sub>t</sub>
180	1.20	0.96	12.2	0.70	24.5	19.5	303.7	1.592	0.188	14.7
181	1.20	1.01	14.7	0.72	17.0	15.5	304.0	1.592	0.222	14.7
182	1.18	1.08	26.5	13.6	5.4	16.7	304.7	1.680	0.0259	14.7
183	1.18	0.97	20.5	13.6	3.5	16.4	304.7	1.680	0.0200	14.7
184	1.18	0.93	20.6	13.5	10.2	18.0	304.6	1.680	0.0204	14.7
185	1.18	0.90	18.3	13.6	15.8	20.8	304.5	1.680	0.0181	14.7
186	1.18	0.90	19.9	13.4	24.0	26.5	304.4	1.680	0.0198	14.7
187	1.18	0.90	23.9	13.4	31.2	31.3	304.2	1.680	0.0233	15.4
188	1.18	0.90	24.0	13.5	36.5	36.0	303.7	1.680	0.0230	16.0
189	1.18	0.90	26.5	13.6	41.1	40.3	303.3	1.680	0.0259	17.2
190	1.18	0.92	11.0	0.72	10.2	15.0	303.4	1.680	0.161	14.7
191	1.18	0.96	11.1	0.71	17.0	15.3	303.8	1.680	0.164	14.7
192	1.18	0.99	13.1	0.72	23.5	18.0	303.8	1.680	0.204	14.7
193	1.18	0.99	13.5	0.71	32.1	24.1	304.0	1.680	0.216	14.7

Table 2. Data for the V/L Ratio Determination by the Concentration Method  
(concluded)

Run No.	$E_s$	$E_t$	$h_s$	$Q_w$	$Q_g$	$P_g$	$T_g$	$N_s$	$N_t$	$P_t$
194	1.18	1.00	13.5	0.72	38.8	30.3	304.0	1.680	0.177	14.7
195	1.18	1.02	10.1	0.72	36.5	33.0	302.7	1.680	0.1354	21.7

Table 3A. Single-Phase Mixing Lengths Observed  
by Injecting Malachite Green into  
Water Flowing in a 1-Inch Glass Pipe  
at 70°F.

$Q_1$ (GPM)	$N_{RE}$	Mixing Length $L/D$
0.85	2,720	80
2.05	6,560	72
2.42	7,750	64
3.30	10,550	60
5.21	16,700	56
9.45	30,200	50
11.5	36,800	50
18.0	57,600	47
27.0	86,400	43
32.5	104,000	45



Table 3B. Single-Phase Mixing Lengths Observed  
by Injecting Malachite Green into  
Water Flowing in a 1 1/2-Inch Glass  
Pipe at 70°F.

$Q_1$ (GPM)	$N_{RE}$	Mixing Length L/D
1.55	3,310	108
2.42	5,160	72
3.28	7,000	60
5.25	11,200	56
7.27	15,500	55.3
9.35	19,950	54
11.60	24,800	48
19.2	41,000	40.6
26.6	56,700	40
32.5	69,400	37.4

Table 3C. Single-Phase Mixing Lengths Observed  
by Injecting Malachite Green into  
Water Flowing in a 1 1/2-Inch Glass  
Pipe at 68°F.

$Q_1$ (GPM)	$N_{RE}$	Mixing Length L/D
1.0	2,130	78.7
1.85	3,950	53.4
1.30	2,780	68
2.97	6,340	36
5.21	11,100	34.8
7.28	15,500	53.4
9.35	19,900	37.3
11.40	24,200	43.4
12.3	26,200	50.6
19.0	40,500	48
23.7	50,600	44
12.3	26,200	48
23.7	50,600	44
27.0	57,600	56
30.1	64,200	56
32.8	70,000	52

Table 3D. Single-Phase Mixing Lengths Observed by Injecting Rosaniline Red into Water Flowing in a 1 1/2-Inch Glass Pipe at 70°F.

$Q_1$ (GPM)	$N_{RE}$	Mixing Length L/D
2.20	4,700	85.5
2.52	5,380	68
2.46	5,250	64
5.21	11,150	60
9.35	19,950	52
9.35	19,950	48
18.0	38,400	44
18.0	38,400	46
26.6	56,700	44
32.5	69,400	40

## APPENDIX II

## ERROR ANALYSIS

The following development gives an expression for the maximum error that can be expected in determining the V/L ratio. The expression is given in terms of the experimentally determined variables.

Equation (17) gives the expression for the V/L ratio in terms of  $v_g$ ,  $v_l$ , C and  $C_o$ . Then the total differential of V/L is

$$d\left(\frac{V}{L}\right) = \frac{\partial\left(\frac{V}{L}\right)}{\partial C_o} dC_o + \frac{\partial\left(\frac{V}{L}\right)}{\partial C} dC + \frac{\partial\left(\frac{V}{L}\right)}{\partial\left(\frac{v_g}{v_l}\right)} d\left(\frac{v_g}{v_l}\right) \quad (26)$$

For the partial differentials one obtains the following expressions from equation (17).

$$\frac{\partial\left(\frac{V}{L}\right)}{\partial C_o} = -\frac{v_g}{v_l} \frac{C}{C_o^2} \quad (27)$$

$$\frac{\partial\left(\frac{V}{L}\right)}{\partial C} = \frac{v_g}{v_l} \frac{1}{C_o} \quad (28)$$

$$\frac{\partial\left(\frac{V}{L}\right)}{\partial\left(\frac{v_g}{v_l}\right)} = \left(\frac{C}{C_o} - 1\right) \quad (29)$$



By substituting equations (27) through (29) in equation (26) and dividing by equation (17) one obtains an expression for the differential error

$$\frac{d\left(\frac{V}{L}\right)}{\left(\frac{V}{L}\right)} = - \frac{dC_o}{C_o} \frac{1}{\left(1 - \frac{C_o}{C}\right)} + \frac{dC}{C} \frac{1}{\left(1 - \frac{C_o}{C}\right)} + \frac{d\left(\frac{V_g}{V_l}\right)}{\left(\frac{V_g}{V_l}\right)} \quad (30)$$

If it is assumed that  $v_1$  is relatively insensitive to temperature and pressure and that the gas follows the perfect gas law, then the differential error in terms of  $C$ ,  $C_o$ ,  $P$  and  $T$  becomes

$$\frac{d\left(\frac{V}{L}\right)}{\left(\frac{V}{L}\right)} = - \frac{dC_o}{C_o} \frac{1}{\left(1 - \frac{C_o}{C}\right)} + \frac{dC}{C} \frac{1}{\left(1 - \frac{C_o}{C}\right)} + \frac{dT}{T} + \frac{dP}{P} \quad (31)$$

All of the variables are measured at the point at which the  $V/L$  ratio is desired except  $C_o$ , which is the concentration based on the total mass flow rate.

Equation (31) may be written in a form that emphasizes the dependence on the mass flow ratio,  $W_g/W_l$ . By rearranging equation (16) one obtains

$$\frac{1}{\left(1 - \frac{C_o}{C}\right)} = 1 + \frac{1}{\left(\frac{W_g}{W_l}\right)} = 1 + \frac{W_l}{W_g} \quad (32)$$

Since the greatest error in  $V/L$  will occur when all the terms act in the same direction, then by using the absolute values of the errors, and substituting equation (32)

into equation (31), the equation for the maximum error is

$$\left[ \frac{d\left(\frac{V}{L}\right)}{\left(\frac{V}{L}\right)} \right]_{\max} = \left(1 + \frac{W_2}{W_3}\right) \left| \frac{dC_o}{C_o} \right| + \left(1 + \frac{W_2}{W_3}\right) \left| \frac{dC}{C} \right| + \left| \frac{dT}{T} \right| + \left| \frac{dP}{P} \right| \quad (23)$$

For a given error in measuring the concentration, the error in the V/L ratio tends toward infinity as the ratio of the mass flow rates,  $W_g/W_l$ , tends toward zero.

If only the effects of the concentration measurement errors are to be considered, the terms for pressure and temperature are not present and the equation for the error in the ratio of the mass flow rates becomes

$$\left[ \frac{d\left(\frac{V}{L}\right)}{\left(\frac{V}{L}\right)} \right]_{\max} = \left[ \frac{d\left(\frac{W_g}{W_l}\right)}{\left(\frac{W_g}{W_l}\right)} \right]_{\max} = \left(1 + \frac{W_2}{W_3}\right) \left| \frac{dC_o}{C_o} \right| + \left(1 + \frac{W_2}{W_3}\right) \left| \frac{dC}{C} \right| \quad (24)$$

## APPENDIX III

## METHOD OF CALCULATION

The calculation of the V/L ratio and associated quantities as presented in Table 1 were made with the aid of the IBM 650 computer of the Rich Electronic Computer Center of the Georgia Institute of Technology. The Bell General Purpose System was used in adapting these calculations to the IBM 650. The program used in making these calculations are on file in the School of Chemical Engineering at the Georgia Institute of Technology.

The program was used to compute the V/L ratio as measured by the concentration method. The input data consisted of the following variables for each run made:

Run Number	
$E_s$	millivolts
$E_t$	millivolts
$h_s$	dimensionless
$Q_w$	GPM
$Q_g$	CFM
$P_g$	psia
$T_g$	$^{\circ}\text{K}$
$N_s$	moles/liter
$N_t$	moles/liter

$P_t$	psia
$\mu_t$	centipoise
$\Delta Q_g$	CFM
$\Delta P_g$	psia
$\Delta T_g$	$^{\circ}\text{K}$
$\Delta h_g$	dimensionless
$\Delta Q_w$	GPM

Delta ( $\Delta$ ) represents the estimated precision of the measurement.

The calculations as indicated by the following equations were made by the computer. Double subscripts indicate that the quantity was computed twice.

$$t_{st} = 25.03 E_{st} \quad (1)$$

$$a_{st} = 0.86332 + 2.2624 \times 10^{-3} t_{st} - 1.046 \times 10^{-5} t_{st}^2 \quad (2)$$

$$b_{st} = 0.86464 + 2.1959 \times 10^{-3} t_{st} - 1.394 \times 10^{-5} t_{st}^2 \quad (3)$$

$$c_{st} = \frac{-b + \sqrt{b^2 + 0.16Na}}{2} \quad (4)$$

$$p_{st} = \frac{b + \sqrt{b^2 + 0.16Na}}{2a} \quad (5)$$



$$W_s = 4.44 \times 10^{-4} h_s \left( 0.89 + \frac{h_s}{450} \right) \left[ \rho_s (7.45 - \rho_s) \right]^{\frac{1}{2}} \quad (6)$$

$$W_l = \frac{Q_w}{7.481} \cdot \frac{P_w}{60} + W_s \quad (7)$$

$$W_g = Q_g \left[ 20 \frac{P_g}{T_g} \right]^{\frac{1}{2}} \frac{492}{530} \cdot \frac{29}{359} \cdot \frac{1}{60} \quad (8)$$

$$C_o = \frac{C_s W_s}{W_l + W_g} \quad (9)$$

$$V_l = \frac{1}{62.43 P_t} \quad (10)$$

$$V_g = 0.666 \left( \frac{273.16 + t_t}{P_t} \right) \quad (11)$$

$$\left( \frac{V}{L} \right)_c = \frac{V_g}{V_l} \left( \frac{C}{C_o} - 1 \right) \quad (12)$$

$$\left( \frac{V}{L} \right)_m = \frac{V_g}{V_l} \left( \frac{W_g}{W_l} \right) \quad (13)$$

$$N_{RE_2} = 2.273 \times 10^4 \left( \frac{W_l}{\mu_t} \right) \quad (14)$$

$$\left[ \frac{\Delta \left( \frac{W_g}{W_l} \right)_m}{\left( \frac{W_g}{W_l} \right)_m}_{est} \right] = \frac{\Delta Q_g}{Q_g} + \frac{\Delta P_g}{P_g} + \frac{\Delta T_g}{T_g} + \frac{0.810 \Delta h_s + \Delta Q_w}{0.810 h_s + Q_w} \quad (15)$$

$$100e = \frac{\left( \frac{V}{L} \right)_m - \left( \frac{V}{L} \right)_c}{\left( \frac{V}{L} \right)_m} \cdot 100 \quad (16)$$

The following quantities were punched out on IBM cards by the computer.

Run Number	
$W_g$	lb./sec.
$(W_g/W_l)_m$	dimensionless
$C/C_0 - 1$	dimensionless
$C$	$\frac{\text{lb. solute}}{\text{lb. solution}}$
$N_{RE1}$	dimensionless
estimated % error in $(W_g/W_l)_m$	dimensionless
$(V/L)_m$	dimensionless
$(V/L)_c$	dimensionless
$e$	dimensionless

Table 4. Viscosity of Sodium Hydroxide Solutions  
(International Critical Tables, Vol. 5, p. 15,  
1929).

Normality	Viscosity of solution at t°C Viscosity of water at t°C	
	18°C	25°C
0.1	1.020	1.023
0.25	1.052	1.055
0.5	1.108	1.110
1	1.234	1.236
2	1.59	
4	2.78	
8	7.04	

## APPENDIX IV

## A PROPOSED METHOD FOR MEASURING CONCENTRATION

It was apparent from the beginning of the investigation that a rapid means of determining the concentration of a sample was necessary and it would be especially useful if a method could be devised whereby the concentration at a point could be determined continuously. A number of principles were considered including measuring the electrical conductivity, the refractive index, the viscosity and the photoconductivity of various solutions. A number of schemes were evolved but the one that showed the most promise was that of measuring the transmission of light through a dye solution.

Although a number of commercial instruments were available to measure these quantities, the cost was prohibitive to this project. The main requirement then became to design an inexpensive method for monitoring the concentration within 1/2 per cent accuracy. The tolerance was established at 1/2 per cent since it was apparent that the mass flow rate of each phase could not be metered to better than 1/2 per cent accuracy. Also, the injection of the dye into the flow system could not be controlled to better than 1/2 per cent accuracy. This meant that the concentration at a point in the system could not be known by a mass balance to better than 1 1/2 per cent.



Accordingly, the following system was designed.

A cell was built composed of inlet and outlet ports for the flowing solution and windows through which a beam of light could pass through the solution. A 2.2-volt light bulb incorporating a focusing lens provided the illumination and a lead storage cell supplied a constant voltage. A Clairex type CL-3 cadmium selenide crystal photocell was the sensing device. A modified Wheatstone bridge arrangement was employed to measure the resistance of the photocell. A complete description of the apparatus is contained in the succeeding paragraphs and schematic drawings are shown in Figs. 12 and 13. Photographs of the device are shown in Figs. 14 and 15.

The Cell.--Two 1/4-inch copper tees were joined at the run by a short piece of 1/4-inch copper tubing and soldered at the joint. Two glass windows, 1/16-inch thick, were fixed in the tees with epoxy resin. These windows were butted as close as possible to the inlet and outlet ports. This cell was then set in a 6 x 8 x 8-inch wooden box with the inlet and outlet ports protruding from the box.

Black rubber tubing was inserted in the tees from the glass windows to the end of the tube to diminish reflected light.

The Light Source.--The light source was a 2.2-volt No. 222 Chicago bulb of the type normally used in a penlight. This bulb had a lens built in although the axis of the bulb did not coincide with the axis of the lens. Accordingly, in

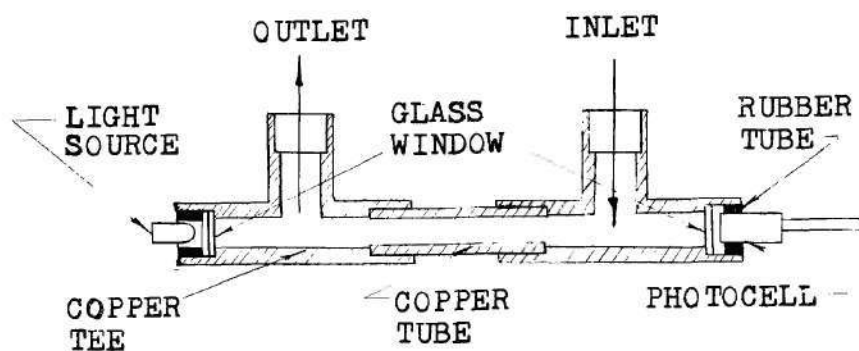


Figure 12. Sketch of Concentration Measuring Cell

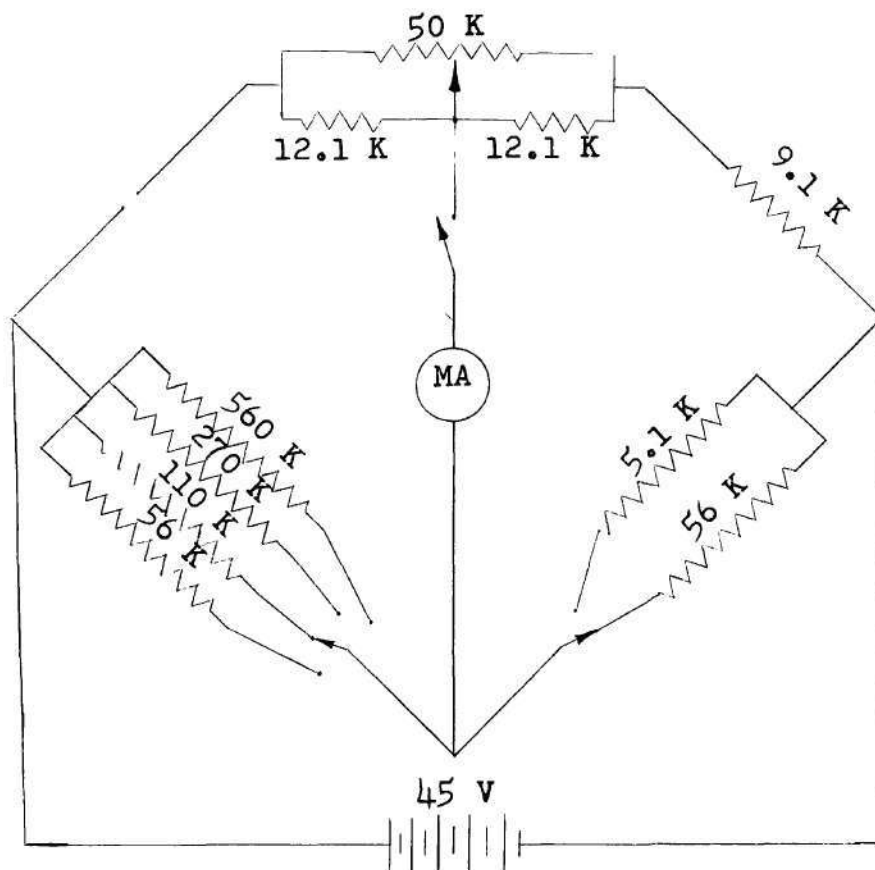


Figure 13. Diagram of Modified Wheatstone Bridge Circuit

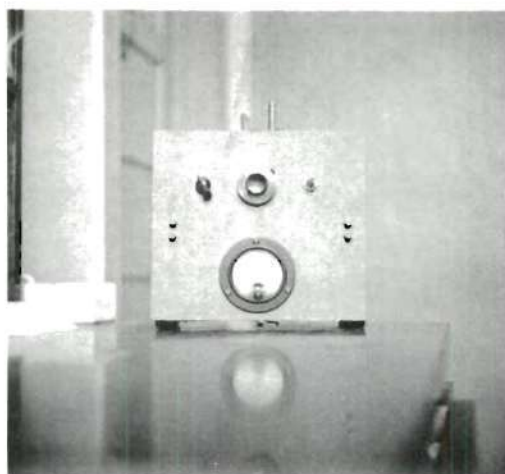


Figure 14. Concentration Measuring Device  
Showing Instruments and Inlet and  
Outlet Lines

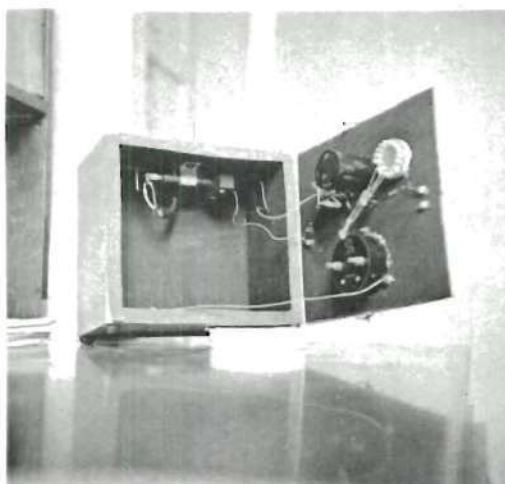


Figure 15. Concentration Measuring Device  
Showing Cell and Wheatstone Bridge

mounting the bulb provisions were made to allow adjusting the axis of the cell. An Allstate storage battery was used to supply the necessary voltage. Only one cell at a time was used which provided a constant voltage of approximately two volts after a declining voltage period of from three to four hours. A variable resistor in series with the light and storage cell permitted adjustment of the voltage across the light bulb.

The Photocell.--A Clairex CL-3 cadmium Selenide crystal photocell was firmly mounted in the end of the cell opposing the light. It was mounted so that the axis of the photocell coincided with the axis of the cell. The leads from the photocell were connected across the measuring section of a Wheatstone bridge circuit.

The Bridge Circuit.--The bridge circuit is shown schematically in Fig. 13. Resistances 1 and 2 could be changed by means of switches to vary the range of the circuit. The circuit was balanced with a 50,000-ohm Helipot variable resistor. Two 12,100-ohm resistors were connected from the centertap to the end terminals of the Helipot. A Weston milliammeter, 30-0-30 ma, indicated a zero reading when the circuit was balanced. This circuit, using a 45-volt potential, could measure resistance from approximately 6,000 to 2,000,000 ohms and could duplicate measurements within 1 per cent precision on the range of 6,000 to 20,000 ohms.

The instrument was used on a comparative basis rather than as an absolute resistance meter. That is, a dye solution



of known concentration was placed in the cell and the resistance in series with the light was varied until the bridge balanced for a preset value on the Helipot. Then with unknown solutions in the cell a balance reading could be obtained on the Helipot.

The dye initially selected was Rosaniline Hydrochloride, a red dye. It was found that the instrument was insensitive to concentration. The data on the Clairex photocell indicated maximum sensitivity in the red and infra-red region of the spectrum so it was presumed that a dye that was a strong absorber of red would give more sensitive readings. Another dye, Victoria Green or Malachite Green, was used and gave excellent results as far as sensitivity was concerned.

The device was tested by passing a standard solution through the cell and adjusting the intensity of the light until the bridge balanced with the potentiometer scale at a standard setting. Then solutions of known concentration were passed through the cell and the potentiometer reading recorded. The results of several attempts at calibration are shown in Table 5. A solution containing 0.0388 grams per liter of Malachite Green was used as the standard solution and the scale reading on the potentiometer was set at 760 for this solution. The second column contains the readings obtained when solutions in the first column were passed through the cell immediately after the instrument was reset with the standard solution. Readings obtained with solutions subse-

quently passed through are shown in the third column. Care was taken to eliminate bubbles and all readings obtained were constant. However, it is apparent that results could not be reproduced. One possible reason is that the design of the cell was faulty such that bubbles could collect and be held in the cell thereby giving false readings. Another possibility is that the glass windows could have become stained although methanol rinses were frequently used and it was observed that methanol removed all apparent traces of dye. It is the opinion of this investigator that through careful design of the cell and a more carefully chosen light source that this instrument can be developed into a practical means of measuring the concentration of a dye over a limited range of concentration.

This project was abandoned due to the limitation of time and another method was adopted to measure the concentration.

Table 5. Potentiometer Scale Readings Versus Concentration of Malachite Green

Concentration of Solution (gm/liter)	Reading of Potentiometer immediately after reset			Reading of potentiometer subsequent to reset				
0.388								
0.194	879	865		872				
0.097				833	823	822		
0.0485	782			801				
0.0388	760	760	760	755	757	759	757	749
					757	763	751	768
0.0194	695	698	681	685				
0.0097	620			670	600	607		
0.00485	510			591	507	512		
0.00388	501	510	499	548	503			
0.00194				436	460	431	452	
0.00097	420			404	424			
0.000485				400				
0.000388	409	401		408	412	401		
0.000194				404	400	401	396	



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